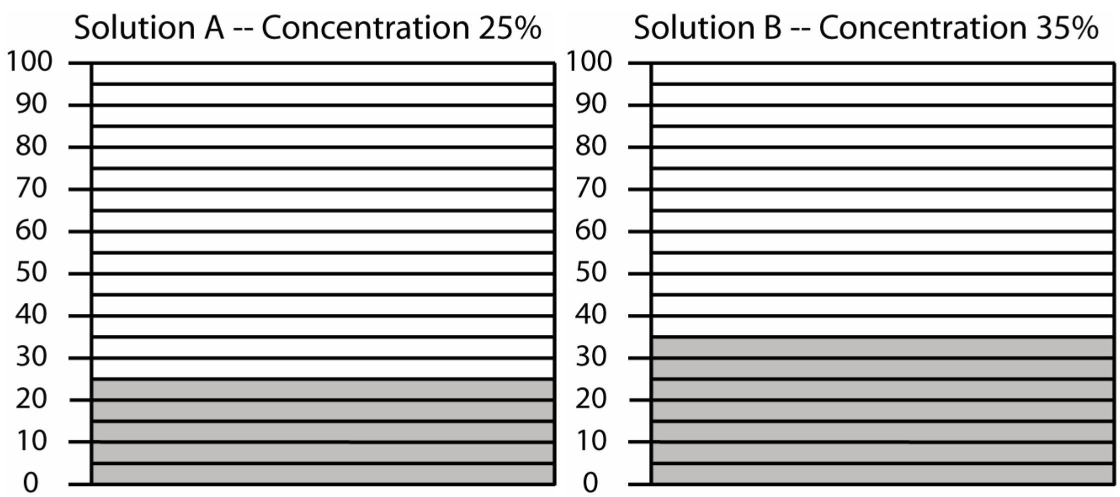
Mixture problems: Salt concentration Solutions NAME:

This worksheet will help us understand mixture problems that involve combining liquids of different concentrations to form a final mixture of another concentration.

Consider two salt solutions, Solution A and Solution B. Solution A is a 25% salt solution and Solution B is a 35% salt solution. This means that out of every 100 gallons of Solution A, there are 25 gallons of pure salt and 75 gallons of water. Likewise, Solution B has 35 gallons of pure salt and 65 gallons of water for every 100 gallons of solution.

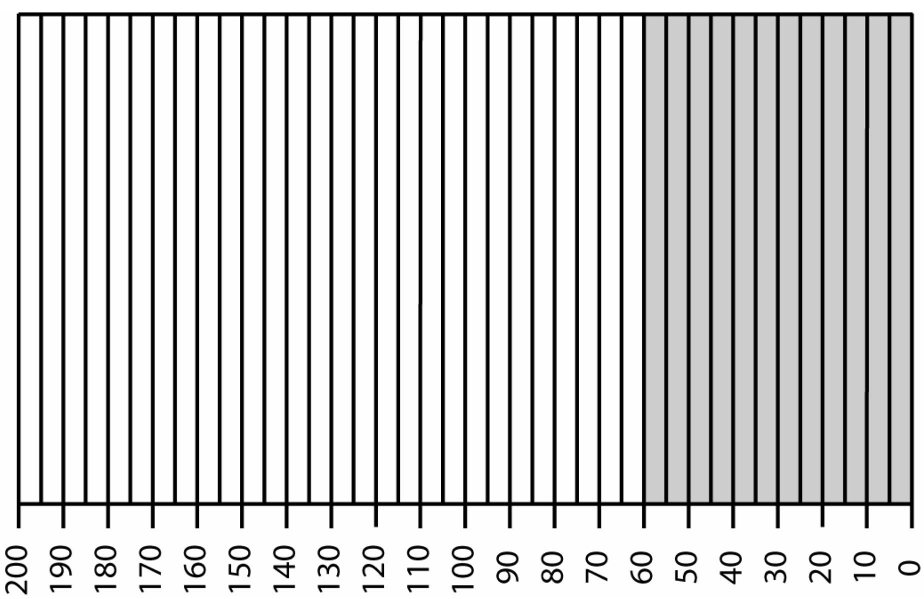
To understand this, I like to picture the salt settling down to the bottom of the jug. The following pictures illustrate two 100-gallon jugs, one with Solution A and one with Solution B. The pure salt has settled to the bottom as the gray material. The white area is pure water.



Let's say we combine these two jugs. We'd make 200 gallons of salt solution. What will the new concentration be? Will it be 25% or 35%? Or neither? Could it be anything?
So, let's combine those two jugs into this bigger jug. Shade in the gallons of pure salt from the two jugs above as if it settled to the bottom of this 200-gallon jug (picture below). What is the final concentration of the mixture?

$$Concentration = 60/200 = 30\%\ Solution$$

Remember the concentration is found by dividing the amount of pure salt by the total amount of stuff (salt and water).



Out of the 100 gallons of Solution A, there were 25 gallons of pure salt; shade that in. Out of the 100 gallons of Solution B, there were 35 gallons of pure salt; shade that in. Together, there are 60 gallons of pure salt in the 200 gallons of stuff (salt and water). Divide 60 by 200 to get the final solution is 30%.

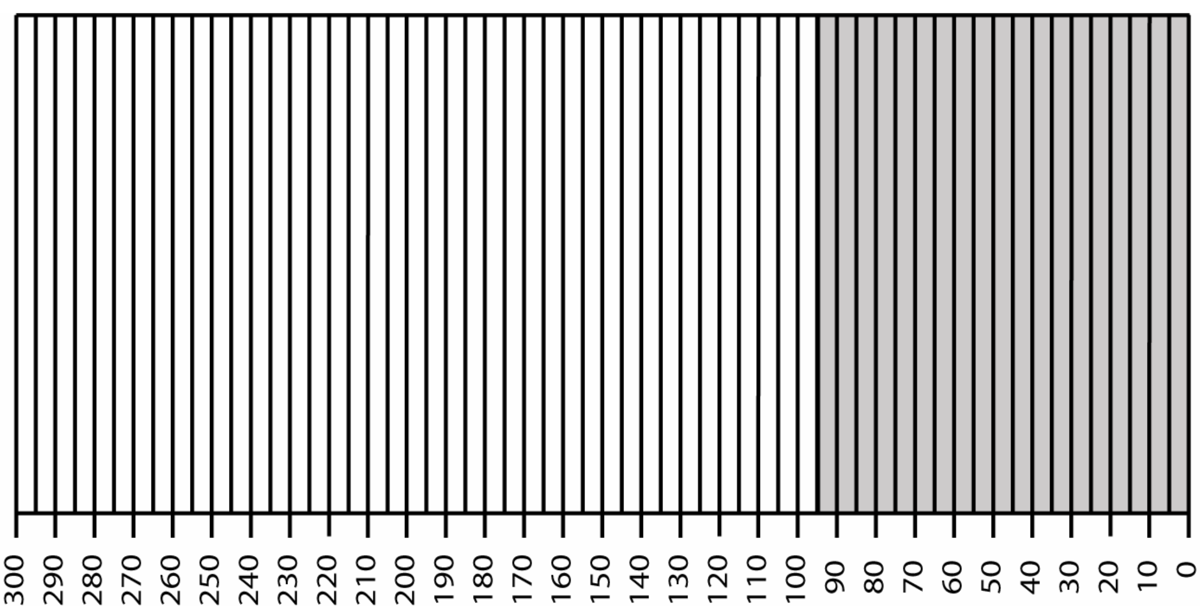
To better understand the relationship between the amounts of each solution we add and the final concentration, we'll play with them a bit.

Let's say we mix 100 gallons of Solution A and 200 gallons of Solution B.

Shade the salt in the 300-gallon jug to the right. Remember to shade twice as much for Solution B since you are adding 200 gallons this time.

Divide the amount of salt by the total amount. What is the concentration?

$$\textit{Concentration} = {}^{95}\!/_{300} = 31.67\%\ \textit{Solution}$$



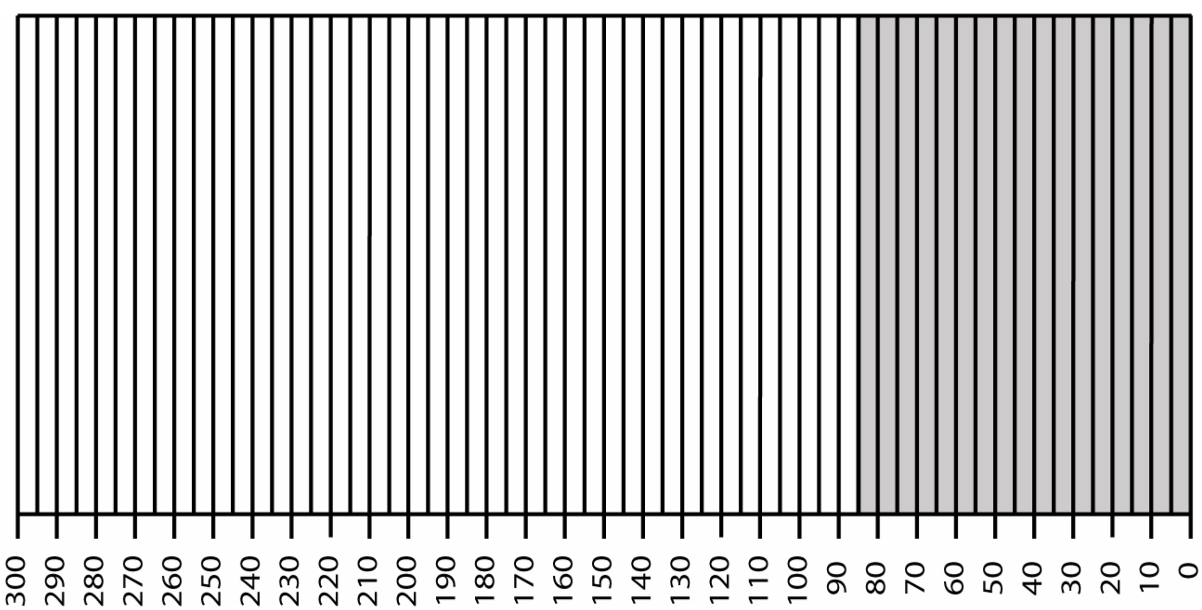
We shade 25 gallons of pure salt for Solution A and 70 gallons of pure salt for Solution B (that's twice 35). This makes 95 gallons of pure salt out of a total of 300 gallons of stuff (salt and water). Divide 95 by 300 to get the concentration of 31.67 %.

Let’s do it again but now with 200 gallons of Solution A and 100 gallons of Solution B.

Shade the salt in the 300-gallon jug to the right. Remember to shade twice as much for Solution A this time since you are adding 200 gallons this time.

Divide the amount of salt by the total amount. What is the concentration?

$$\textit{Concentration} = {}^{85}\!/_{300} = 28.33\%\ \textit{Solution}$$



We shade 50 gallons of pure salt for Solution A (that’s twice 25) and 35 gallons of pure salt for Solution B. This makes 85 gallons of pure salt out of a total of 300 gallons of stuff (salt and water). Divide 85 by 300 to get the concentration of 28.33 %.

Isn’t this fun?

YES! YES! YES!

Discuss how the final concentration varies as the amounts of each solution vary. Complete the table to help you organize the facts.

Gallons of Solution A	**Gallons of Solution B**	**Final concentration**
100	100	*30 %*
100	200	*31.67 %*
200	100	*28.33 %*

When you add equal amounts of both solutions, the final concentration is the exact middle of the two solutions' concentrations. If you add more Solution A than Solution B, the final concentration is closer to the concentration of Solution A. If you add more Solution B than Solution A, the final concentration is closer to the concentration of Solution B.

These next few problems are too large for pictures but use the same reasoning as before. Figure the final concentrations.

Gallons of Solution A	**Gallons of Solution B**	**Final concentration**
100	2000 (That's 20 jugs.)	*34.52 %*
2000 (That's 20 jugs.)	100	*25.48 %*

$$\frac{25+700}{2100} = 34.52\%$$

$$20(35) = 700$$
$$20(25) = 500$$

$$\frac{500+35}{2100} = 25.48\%$$

Estimate upper and lower bounds for the concentration of the final mixture. In other words, there are limits on what the concentration of the final mixture can be. What are those limits? The calculations we have done here will help.

We added much, much more Solution B than A (first row) to get 34.52%. Notice this got the concentration to be nearly 35%, the concentration of Solution B. That 35% is the upper limit for the final concentration. It will never go higher than 35%. Try it out! Say you add 40,000 gallons of Solution B; what is the final concentration?

Likewise, we added much, much more Solution A than B (second row) to get 25.48%. This 25% is a lower limit for the final concentration. This idea is similar to what the book discusses in problem 75, page 141.

Let’s use what we have learned here to solve a typical mixture problem.

We have 40 gallons of a 25% salt solution. How much 80% solution must be added so that the final mixture is 40% solution?

(Let x represent the number of gallons of 80% solution we add. Then figure an expression for the final mixture’s concentration using x. Remember the final mixture’s concentration will be the amount of pure salt divided by the total amount of stuff (water and salt). Set this expression equal to .40 and solve.)

Let x represent the number of gallons of 80% solution we add.

The verbal model is below.

$$\boxed{\textit{Concentration of final mixture}} = \frac{\boxed{\textit{Amount of pure salt in final mixture}}}{\boxed{\textit{Total amount of stuff (salt and water) in final mixture}}}$$

So we put the pieces where they go. The final concentration will be .4. The amount of pure salt in the final mixture is the amount of pure salt from the 25 % solution plus the amount of pure salt we add. In the 40 gallons of 25 % solution, there are 10 gallons of pure salt. In the x gallons of 80% solution, there are .8x gallons of pure salt. The total amount of stuff (salt and water) has to be the 40 plus the x we add.

$$.4 = \frac{10 + .8x}{40 + x}$$

$$.4(40 + x) = 10 + .8x$$

$$16 + .4x = 10 + .8x$$

$$6 = .4x$$

$$15 = x$$

Use algebra to fill in the pieces of the verbal model. Multiply both sides by the denominator on the right to simplify things. Then distribute the .4 on the left and isolate x.

So, we add 15 gallons of 80% solution to our 40 gallons of 25% solution in order to make the 40% solution in the end.