Solving absolute value and radical equations Solutions                    NAME:

This worksheet is designed to help you make sense of some of the methods we use to solve absolute value and radical equations algebraically. We will also look at solving these equations graphically.

1a. Consider the equation $|4x-2|=7$. If this equation is true, what must be true of the number $4x-2$? Why? (In other words, what two numbers could this number, $4x-2$, possibly be? How do you know for sure?)

*Absolute value is distance to zero. Saying the absolute value of 4x – 2 equals 7 means it is 7 units from zero. This number, 4x – 2, must be either +7 or -7.*

1b. Use what we know about the number $4x-2$ to rewrite $|4x-2|=7$ as two equations. (Notice the absolute value signs are gone at this point.) Then solve them to find the two solutions to $|4x-2|=7$.

$4x-2=7$   *or*   $4x-2=-7$

$4x=9$   $4x=-5$

$x=9/4$   $x=-5/4$

$x=2.25$   $x=-1.25$

1c. Check your two solutions. Do they make the original equation true?

$|4x-2|=7$   $|4x-2|=7$

$|4(2.25)-2|\stackrel{?}{=}7$   $|4(-1.25)-2|\stackrel{?}{=}7$

$|9-2|\stackrel{?}{=}7$   $|-5-2|\stackrel{?}{=}7$

*yes*!   *yes*!

*Both solutions make the original equation true. They are solutions.*

2. Solve the equation $|6+3x|-4=10$. If you do it algebraically, isolate the absolute value part before you use the procedure above and show your work. If you do it graphically, draw a labeled, complete graph with the solution labeled.

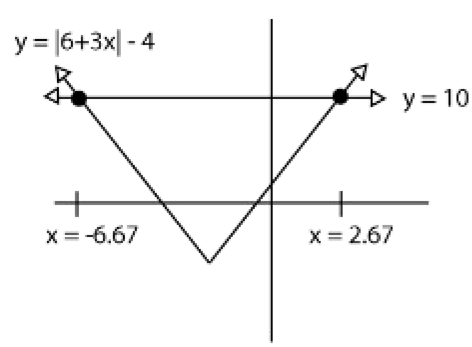
$|6+3x|-4=10$

$|6+3x|=14$

$6+3x=14$   *or*   $6+3x=-14$

$3x=8$   $3x=-20$

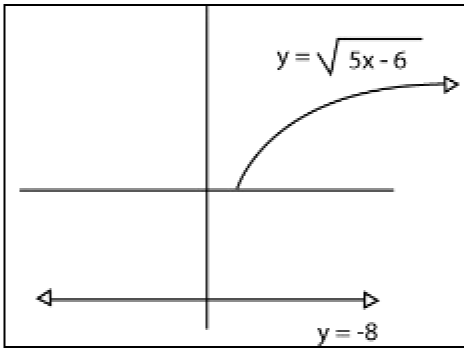
$x=2.67$   $x=-6.67$



3a. Why must the equation $\sqrt{5x-6}=-8$ have no solution? (In other words, what about the equation makes it so that there is no $x$ value that makes it true?)

> *You cannot take the square root of a number and get a negative number. Square roots are always positive. There is no number whose square root is -8. Therefore there is no solution to the equation.*

3b. Draw the labeled, complete graph that would be used to solve the equation $\sqrt{5x-6}=-8$. From the graph, how can you tell that there is no solution to the equation?

| | |
|---|---|
|  | *There is no solution to this equation because the left and right sides do not intersect.* |

3c. Often, people will attempt to solve $\sqrt{5x-6}=-8$ algebraically, not realizing that the equation does not work. Below, you will see the (wrong) solution that is most common. Check the solution given below in the original equation to show that it does not work.

$$\sqrt{5x-6}=-8$$
$$\left(\sqrt{5x-6}\right)^2=(-8)^2$$
$$5x-6=64$$
$$5x=70$$
$$\boxed{x=14}$$

$$\sqrt{5x-6}=-8$$
$$\sqrt{5(14)-6}\overset{?}{=}-8$$
$$\sqrt{64}\overset{?}{=}-8$$

*No No No* $\sqrt{64}=8$

*Put 14 into the equation. We see that it does not make the equation true. The square root of 64 is not -8, it's +8. So 14 cannot be a solution to the equation.*

4a. In contrast, the equation $\sqrt{5x-6}=8$ does have a solution. How does it differ from the last equation? Why does that make a difference?

*Here, we have the square root of a number equals +8. This can work. There is a number whose square root is +8. We can find this number.*

4b. Solve the equation $\sqrt{5x-6}=8$ algebraically. Notice you are undoing what was done to the $x$. Show your work. Check your answer by substituting it into the original equation.

$\sqrt{5x-6}=8$

$5x-6=64$

$5x=70$

$x=14$

*We square both sides to undo the square root. Then add 6 and divide by 5 to get 14. We check it on the right.*

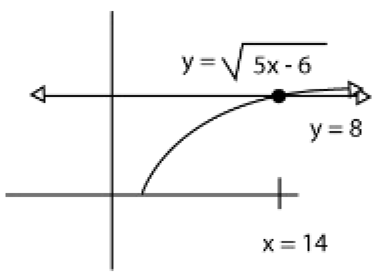
$\sqrt{5x-6}=8$

$\sqrt{5(14)-6}\overset{?}{=}8$

$\sqrt{64}\overset{?}{=}8$

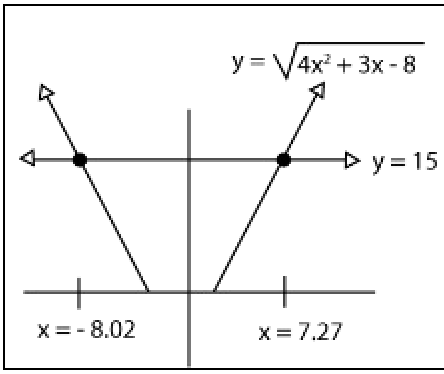
*Yes*

4c. Solve the equation $\sqrt{5x-6}=8$ graphically. Draw a labeled, complete graph with the solution labeled.




*The solution is where the two sides intersect. The only intersection is at 14.*

5. Solve the equation $\sqrt{4x^2+3x-8}=15$. It would probably be easiest to solve it graphically. Draw a labeled, complete graph with the solutions labeled. (An algebraic solution will be an option when we learn the quadratic formula.)




*To see the horizontal line, start on the Standard window and increase your ymax to something above 15. The intersections occur at -8.02 and 7.27. If you subtract 15 first and graph* $y=\sqrt{4x^2+3x-8}-15$, *you would be looking for the x-intercepts. Remember to put parentheses around the expression under the radical.*