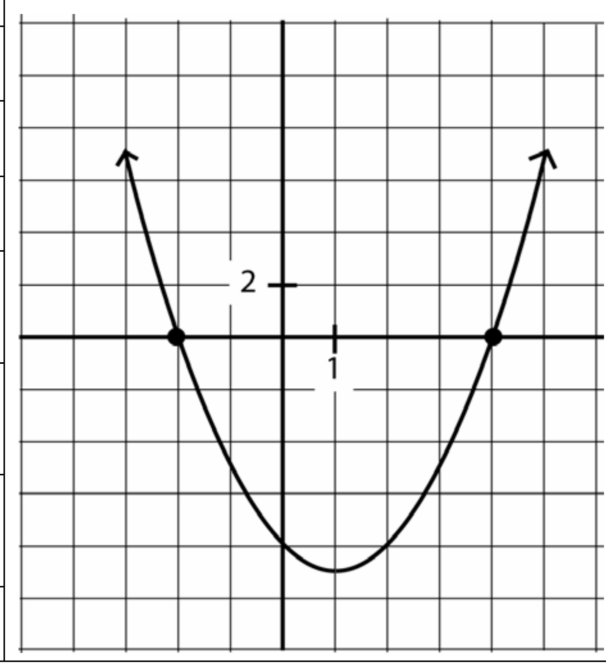
Making Connections Solutions NAME:
Algebraic and graphical solutions

Show your work. Proofread your written answers and circle your numerical answers.

1. Complete the table and use it to sketch the graph of $y = x^2 - 2x - 8$. Notice the scale on the y-axis is two units per tick mark. Also, the vertex (where it turns) is at the point (1, -9). Try to draw your graph as a curved U shape and put arrows on the ends. The x-intercepts will prove to be very important.

x	$h(x) = x^2 - 2x - 8$
-3	*7*
-2	*0*
0	*-8*
1	*-9*
2	*-8*
4	*0*
5	*7*



2. Now solve $0 = x^2 - 2x - 8$ algebraically.
(Hint: Notice the left side is zero, so if you factored the right side into "something times something", you could then set each of the factors equal to zero. Make sure you know why this works.)

$$0 = x^2 - 2x - 8$$
$$0 = (x+2)(x-4)$$
$$x + 2 = 0 \qquad x - 4 = 0$$
$$x = -2 \qquad x = 4$$

3. Now, we solved $0 = x^2 - 2x - 8$ to find the values of x that make the expression $x^2 - 2x - 8$ equal to zero. (Make sure you understand this.) Notice these values are the x-intercepts of the graph of $y = x^2 - 2x - 8$. Is that just a coincidence? Explain.

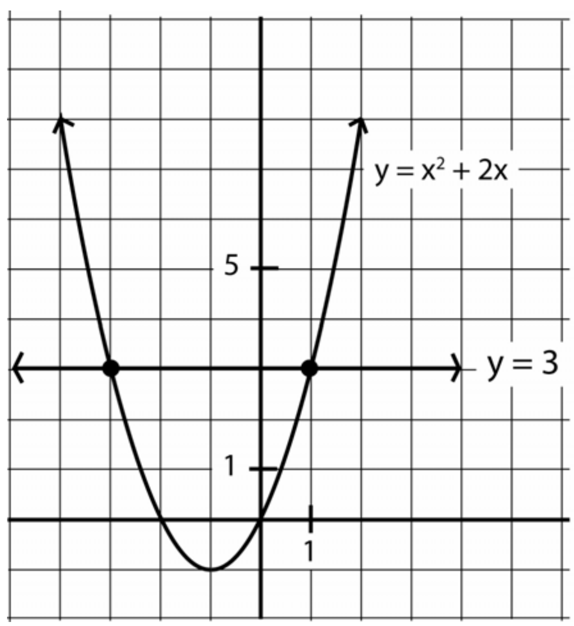
Not a coincidence. To find the x-intercepts of $y = x^2 - 2x - 8$*, we would plug 0 in for y and solve. But that's exactly* $0 = x^2 - 2x - 8$*. So we see that the solutions to 0 = "blah blah blah" are the x-intercepts of y = "blah blah blah".*

4. Let's move on to a different problem. Solve $x^2 + 2x = 3$ algebraically.
(Hint: If you factored the left side now, it would not get you anywhere because you can't set each factor to 3. Why not? So, let's subtract 3 from both sides to get zero on the right, then try to factor. Do you see why this works better?)

$$x^2 + 2x = 3$$
$$x^2 + 2x - 3 = 0$$
$$(x-1)(x+3) = 0$$
$$x - 1 = 0 \qquad x + 3 = 0$$
$$x = 1 \qquad x = -3$$

5. Now we'll graph $y = x^2 + 2x$ and $y = 3$. Complete the table below to graph $y = x^2 + 2x$. The vertex (where it turns) is at the point (-1, -1). Try to draw your graph as a curved U shape and put arrows on the ends. The graph of $y = 3$ is the horizontal line at the *y* value of 3.

x	$y = x^2 + 2x$
-4	*8*
-3	*3*
-2	*0*
-1	*-1*
0	*0*
1	*3*
2	*8*



6. What do you notice about this graph and the solutions to the equation $x^2 + 2x = 3$? (Think about what we're doing. When we solve $x^2 + 2x = 3$, we are finding the *x* values that make the expression $x^2 + 2x$ equal to 3. How do the solutions show up on the graph of $y = x^2 + 2x$ and $y = 3$? Try to make sense of this in your head.)

The graphs of $y = x^2 + 2x$ and $y = 3$ intersect at the x values of -3 and 1. These are the solutions to the equation $x^2 + 2x = 3$.

7. The true beauty of the connection between an equation's solution and the graph of the corresponding function is that it helps us solve extremely difficult equations, ones we could not solve algebraically, like the one required here.

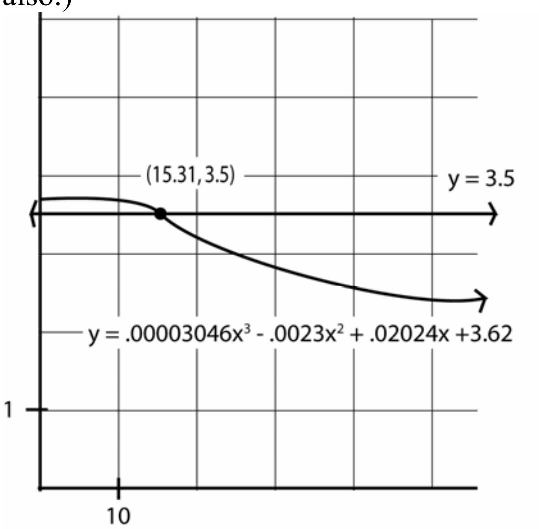
According to data from the US Bureau of the Census, the approximate population of Chicago y (in millions) between 1950 and 2000 is given by $y = .00003046x^3 - .0023x^2 + .02024x + 3.62$ where x is the number of years since 1950. Answer the following questions.

a.) Write an equation that you would use to answer the question "In what year was the population 3.5 million?"

$$3.5 = .00003046x^3 - .0023x^2 + .02024x + 3.62$$

You need to keep straight what x and y represent. The problem says that y is the population and x is time. Since we are given a population value and asked for the time, we know y is 3.5 and x is what we are solving for. Since y is in millions, we do not need 3.5 ***million*** *on the left, just 3.5.*

b.) Sketch a graph of the function $y = .00003046x^3 - .0023x^2 + .02024x + 3.62$ and the horizontal line $y = 3.5$. Use the intersection function on your calculator so you can plot the point of intersection accurately. Label the point of intersection in ordered pair notation clearly on your graph. (The scale of the graph below is [0, 50] x [0, 5]. Set your graph window to that so it's easy to copy. If you set *Xscl* to 10, the tick marks will match also.)

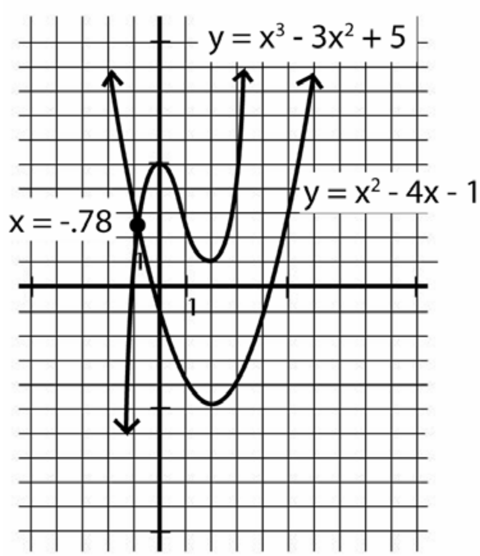


c.) What is the x value of the intersection? Write a sentence expressing the answer to the question "In what year was the population 3.5 million?"

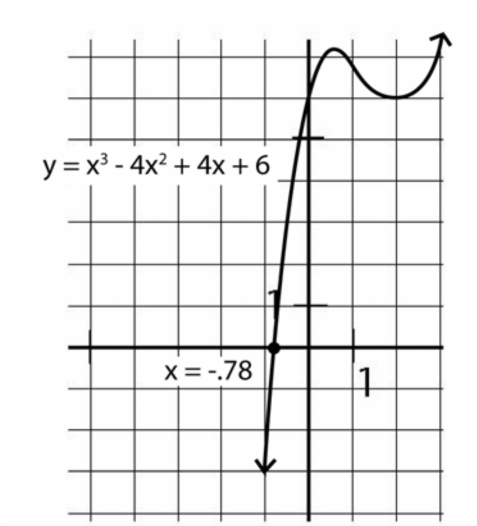
The x value of the intersection is 15.31. This means that the population of Chicago was 3.5 million about 15 years past 1950 or in 1965.

8. Now we'll solve $x^3 - 3x^2 + 5 = x^2 - 4x - 1$. It would be insane to try to solve it algebraically. We have two options to solve it graphically. We'll do both here. You can decide which you like best.

a.) Option 1: Graph both $y = x^3 - 3x^2 + 5$ and $y = x^2 - 4x - 1$ and see where they intersect. Try this method and label the point of intersection with its *x* value. Match your window to the graph below. Use the Intersection function on your calculator, not just TRACE.



b.) Option 2: Set one side to zero – we'll subtract the right side from the left and get $x^3 - 4x^2 + 4x + 6 = 0$ (Make sure this is correct.) Then graph the appropriate graph. What function are you going to graph? What are we looking for on our graph? Label the *x* value of the point which corresponds to the solution to $x^3 - 4x^2 + 4x + 6 = 0$.



We graph $y = x^3 - 4x^2 + 4x + 6$ and look for where y is 0. That means we want the x-intercepts of the graph. So the solution is x = -.78. Notice this is the same as we found above with the other method.

c.) Which option did you prefer? Use whichever method you want from now on.

Sometimes one method may be easier than another. If you are trying one method and not getting anywhere, try the other method. But you should always get the same answer no matter which method you use.