## Discriminants and $x$-intercepts Solutions

NAME:
There are three possibilities for the number of $x$-intercepts of a quadratic function: two, one, or zero. Fill in the following table to develop examples for these three possibilities. Choose small enough values for $\mathbf{a}, \mathbf{b}$, and $\mathbf{c}$ so that you can do the operations in your head.

There are many correct answers here.

| Function | $\begin{gathered} \text { Discriminant } \\ \mathbf{b}^{2}-4 \mathbf{a c} \end{gathered}$ | Graph | Number of $x$-intercepts |
| :---: | :---: | :---: | :---: |
| $y=4 x^{2}+4 x+1$ | $\begin{aligned} & \mathbf{b}^{2}-\mathbf{4 a c}=\mathbf{0} \\ & b^{2}-4 a c \\ & =4^{2}-4(4)(1) \\ & =16-16 \\ & =0 \end{aligned}$ |  | 1 |
| $y=1 x^{2}+4 x+5$ | $\begin{aligned} & \mathbf{b}^{2}-4 \mathbf{a c}<\mathbf{0} \\ & b^{2}-4 a c \\ &=4^{2}-4(1)(5) \\ &= 16-20 \\ &=-4<0 \end{aligned}$ |  | None |
| $y=4 x^{2}+6 x+2$ | $\begin{gathered} \mathbf{b}^{2}-4 \mathbf{a c}>\mathbf{0} \\ b^{2}-4 a c \\ =6^{2}-4(4)(2) \\ =36-32 \\ =4>0 \end{gathered}$ |  | 2 |

1. To form a function that will guarantee $\mathbf{b}^{2}-\mathbf{4 a c}=\mathbf{0}$, do the following.

Select $\mathbf{b}$ to be an even number. Then divide $\mathbf{b}^{2}$ by $\mathbf{4}$. Choose $\mathbf{a}$ and $\mathbf{c}$ so that their product is equal to the quotient $\frac{b^{2}}{4} \cdot \begin{aligned} & \text { I chose } 4 \text { for } b \text {. So } b^{2} \text { is } 16 \text {. Divide that by } 4 \text { and get } 4 . \\ & \text { I chose a and c so that their product was } 4,4 \text { and } 1 .\end{aligned}$
2. To form a function that will guarantee $\mathbf{b}^{2}-\mathbf{4 a c}<\mathbf{0}$, do the following.

Select $\mathbf{b}$ to be an even number. Then divide $\mathbf{b}^{2}$ by 4 . Choose $\mathbf{a}$ and $\mathbf{c}$ so that their product is greater than the quotient $\frac{b^{2}}{4} \cdot$| $I$ chose 4 for $b$. So $b^{2}$ is 16 . Divide that by 4 and get 4 . |
| :--- |
| I chose a and $c$ so that their product was greater than |
| 4,1 and 5. |

3. To form a function that will guarantee $\mathbf{b}^{2}-\mathbf{4 a c}>\mathbf{0}$, do the following.

Select $\mathbf{b}$ to be an even number. Then divide $\mathbf{b}^{2}$ by 4 . Choose $\mathbf{a}$ and $\mathbf{c}$ so that their product is less than the quotient $\frac{b^{2}}{4}$. $\begin{aligned} & \text { I chose } 6 \text { for } \text { b. So } b^{2} \text { is } 36 \text {. Divide that by } 4 \text { and get } 9 \text {. } \\ & \text { I chose a and c so that their product was less than } 9,4 \\ & \text { and 2. }\end{aligned}$
4. For each function, calculate $\mathbf{b}^{2}-4 \mathbf{a c}$ in the second column, graph the function in the third column (standard window should be fine), and denote the number of $x$ intercepts in the fourth column.

