Factoring trinomials **NAME:**
Part 1: Introduction, Factoring by grouping, and A-C Method

In general, we are factoring $ax^2 + bx + c$ where *a*, *b*, and *c* are real numbers.

This set of worksheets will show you three different methods. They are the A-C method, the Reverse FOIL method, and the Cross-product method. The A-C method uses factoring by grouping and so that is reviewed on page 3. The Reverse FOIL and Cross-product methods are essentially the same but what you write is different.

To factor an expression means to write it as a product of factors instead of a sum of terms. The expression $3x^2 + 10x + 8$ can be thought of as the sum of three terms. The equivalent expression $(1x + 2)(3x + 4)$ can be thought of as the product of two factors.

This worksheet will work on our ability to factor trinomials, expressions with three terms like $3x^2 + 10x + 8$. To better understand how we factor this expression, we will start by looking at how two binomials (expressions with two terms) multiply together. Usually, the product of two binomials is a trinomial. Factoring is simply going the opposite way.

Let's say we multiply the following two binomials. I will write the steps of **FOIL** out explicitly. I labeled the lines so I can refer to them later.

$(1x + 2)(3x + 4)$ line 1

$= 3x^2 + 4x + 6x + 8$ line 2

F O I L

$= 3x^2 + 10x + 8$ line 3

The A-C method:

Remember this expression $3x^2 + 10x + 8$ is in the general form $ax^2 + bx + c$. It is this *a* and *c* to which the A-C method refers. We will use this specific example and others to make sense of the A-C method.

This is the key to the whole thing. Notice, on line 2 above, if we multiply the 3 (of $3x^2$) and the 8 (the constant term), we would get 24. And, if we multiply the 4 and the 6 (of $4x + 6x$), we would get that same 24. We are seeing that the product of the **F** and **L** coefficients equals the product of the **O** and **I** coefficients. Do you think this pattern would hold for any FOIL problem? We will investigate that next.

Another thing to keep in mind is how the $4x + 6x$ (line 2) added to make the $10x$ in line 3. We will deal with that as well.

Let's look at a few more examples. Find each product by using FOIL. Write out all four terms of FOIL and then combine like terms to end up with a trinomial in each case.

a.) $(x+2)(x+3)$

b.) $(2x+4)(x+3)$

c.) $(2x-3)(x-3)$

Notice in each case, the product of the **F** and **L** coefficients equals the product of the **O** and **I** coefficients. Also, the **O** and **I** terms add to form the middle term of the trinomial. Again, it is these patterns that we latch onto with the A-C method.

The A-C method will tell us to multiply *a* and *c* of our trinomial (ax^2+bx+c), and then find two numbers whose product is the same but also add to *b*. We will use this knowledge to rewrite the *bx* term inside our trinomial. This will result in a four-term expression, which we will then factor by grouping.

For instance, for the trinomial $3x^2+10x+8$, we would rewrite the 10*x* term as 4*x* + 6*x*. That means our trinomial is turned into $3x^2+4x+6x+8$. We would then factor by grouping which is described on the next page.

Remember, factoring and FOIL go opposite ways as shown below. A factoring problem is simply asking for what you would FOIL out to get the trinomial they gave you.

FOIL

$$(1x+2)(3x+4)$$
$$=3x^2+4x+6x+8$$
$$=3x^2+10x+8$$

FACTOR

Factoring by Grouping:
Factoring by grouping means just that. We group the first two terms and factor something out of them. Then we group the last two terms and factor something out of them. If we do it right, what is left over from the first two terms will be the same as what's left over from the last two terms. Here is an example.

Factor $4x$ from $8x^2 + 4x$

Factor –3 from – 6x – 3

$$8x^2 + 4x - 6x - 3$$
$$= 4x(\underline{\underline{2x+1}}) - 3(\underline{\underline{2x+1}})$$
$$= (4x-3)(2x+1)$$

Notice the common factor of $2x + 1$ is left over. I've underlined it to show it off. Factor it out.

Explaining what was done:
Notice how we started with four terms. From the first two terms, we factored out $4x$. From the last two terms, we factored out -3.

This gave us two terms (they are $4x(2x + 1)$ and $3(2x + 1)$) that have a common factor of $2x + 1$. We then factored the $2x + 1$ out and got $(4x – 3)(2x + 1)$.

Notice this results in the factored form of $8x^2 + 4x - 6x - 3$ or $(4x-3)(2x+1)$.

Practice by factoring the following expression by grouping.
$2x^2 - 6x - 3x + 9$

Check your answer by looking at problem *c* on page 2.

The A-C method:

The A-C method tells us to multiply *a* and *c* of our trinomial ($ax^2 + bx + c$), and then find two numbers whose product is the same but also add to *b*. We will use this knowledge to rewrite the *bx* term inside our trinomial. This will result in a four-term expression, which we will then factor by grouping.

Let's try factoring $3x^2 + 4x - 4$. We need two numbers that multiply to make 3(-4) or –12 and add to make 4. Think of the possible factors of –12.

Possible factors of -12		
-1 and 12	-2 and 6	-3 and 4
-12 and 1	-6 and 2	-4 and 3

There is only one pair that adds to 4. So we'll write

$$3x^2 + 4x - 4$$
$$= 3x^2 - 2x + 6x - 4$$

We could also have written $3x^2 + 6x - 2x - 4$. How would that change things?

Now we can finish by factoring by grouping. I did the work below. See the previous discussion of factoring by grouping if you cannot follow it.

$$3x^2 - 2x + 6x - 4$$
$$= x(3x - 2) + 2(3x - 2)$$
$$= (x + 2)(3x - 2)$$

We pull out a common factor from the first two terms (x) and a common factor from the last two terms (2).

This writes the sum we started with as a product of two numbers, $x + 2$ and $3x - 2$.

Try this guided example on your own.
Factor $3x^2 - 11x + 10$. Start by finding the possible factors of 3(10) or 30. (Do not forget the negatives.)

Possible factors of 30			

Now choose the pair that adds to –11 and rewrite the trinomial with four terms.

Now factor by grouping and finish with the factored form of $3x^2 - 11x + 10$. Circle this final answer. You may want to FOIL it out in your head or on scratch paper to check your answer.

Use the A-C method to factor the following. It is always wise to first factor out a GCF from **all** terms if one exists.

a.) $10x^2 + 4x - 6$

b.) $12x^2 - 7x - 10$

c.) $2x^2 + 10x + 12$

d.) $150x^2 - 150x - 36$

e.) $-12x^2 - 23x - 10$

f.) $6x^2 + 7x + 2$