Inequalities and you 3 Solutions NAME:

This worksheet will provide practice for solving absolute value, polynomial, and rational inequalities. We will also work on understanding why the procedures work. We will solve problems both algebraically and graphically.

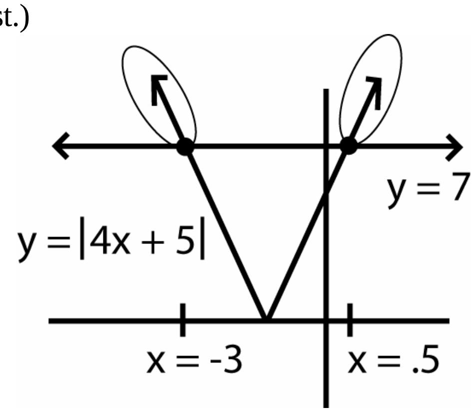
1. Solve the inequality $|4x+5| \geq 7$ **algebraically**. Write your solution in interval notation. (Hint: Remember the absolute value of a number is its distance to zero. If we say the absolute value of a number is greater than or equal to seven, then the number is at least seven units from zero. Considering this, where must the number $4x + 5$ lie? We use this information to set up two separate inequalities which we can then solve.)

$$\begin{array}{lll} 4x+5 \leq -7 & or & 4x+5 \geq 7 \\ 4x \leq -12 & & 4x \geq 2 \\ x \leq -3 & & x \geq .5 \end{array}$$

If this number, call it 4x + 5, is at least 7 units from zero, then it must be left of -7 or right of +7. Notice we use this to write the absolute value inequality as two regular inequalities. Solve these to get the solution. In interval notation, the solution is (-∞, -3] and [.5, ∞). Notice we include the endpoints -3 and .5 because of the "equal to" part of the inequalities.

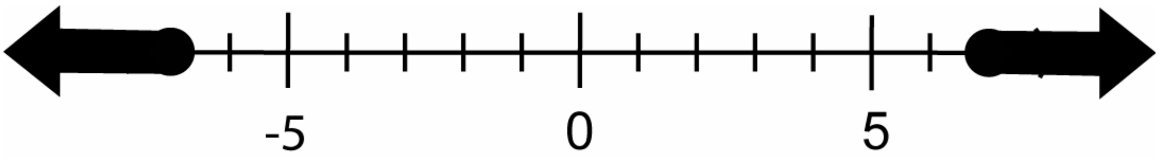
2. Now solve the inequality $|4x+5| \geq 7$ **graphically**. Provide an appropriate graph. Label exactly what you have graphed. Circle the part of the graph that indicates the solution, as I have done in the notes. Does your solution match the algebraic one? (Make sure to label the intersections or x-intercepts, whichever is important to your solution.)

(Calculator usage: On the **TI82**, the second function of the $\boldsymbol{x^{-1}}$ button on the left, labeled **ABS**, is absolute value. On the **TI83**, press the **MATH** button, arrow over to **NUM**, and then select **abs(** for absolute value. On the **TI85** or **86**, enter the **MATH** menu, which is the second function of the multiplication button. Then select **NUM**, and select **abs** from this list.)



*I used the standard window. Here, we are looking for where the V-shaped graph is above or intersecting the horizontal line. We have **equality** at the intersection points. The absolute value of 4x + 5 is **greater** than 7 where the V-shaped graph is **above** the horizontal line. So we need to denote both the intersections -3 and .5 as well as those x values to the left of -3 and to the right of .5. This matches the solution we found algebraically.*

3. I want to investigate why we solve absolute value inequalities the way we do. Think about the inequality $|x| \geq 7$. In words, what does this inequality tell you about the number x? Shade the two portions of the real number line where x might lie. Then use two inequalities (without absolute values) to describe these two portions of the real number line.



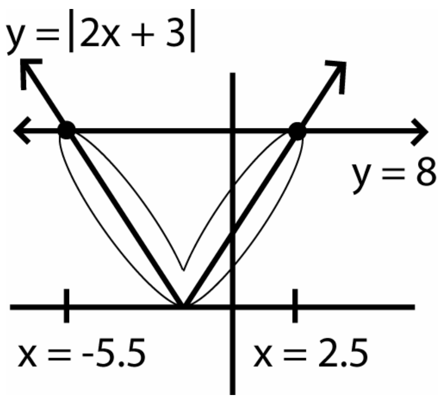
In words, $|x| \geq 7$ is saying that the number x is at least 7 units from zero on the real number line. That is equivalent to saying that x is less than or equal to -7 or greater than or equal to +7. In inequality notation, this is written $x \leq -7$ or $x \geq 7$.

We use this concept when we algebraically solve inequalities like the one in problem #1. The only difference is that we are not dealing with the number x, but rather the number "$4x + 5$". So, we translate $|4x+5| \geq 7$ to mean $4x+5 \leq -7$ or $4x+5 \geq 7$.
Once the absolute value symbols are gone, it becomes easier to solve for x.

4. Let's work with this concept again, but now we'll solve a "less than" problem. Let's say we want to solve the inequality $|2x+3| < 8$. Use the notion of absolute value to rewrite this without absolute value symbols. Then solve **algebraically**. (Notice, here we are dealing with "less than". So the number "$2x + 3$" is **less than** 8 units from zero. How does that change things?)

We are saying the distance the number "2x + 3" is from zero is less than 8. So the number "2x + 3" must be between -8 and +8. So we write and solve the inequality $-8 < 2x+3 < 8$. We subtract 3 and divide by 2 to get the solution $-5.5 < x < 2.5$. In interval notation, this would be (-5.5, 2.5).

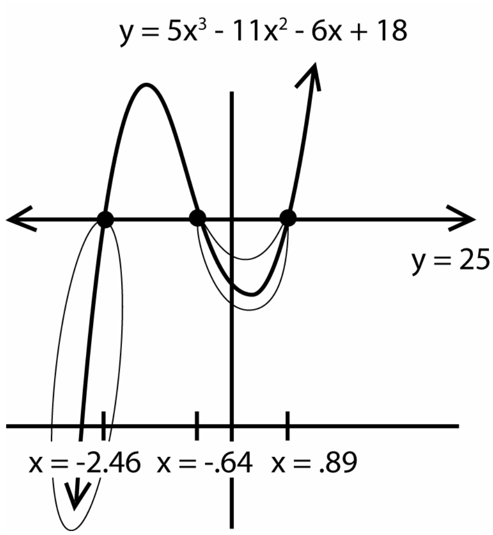
5. Solve $|2x+3|<8$ **graphically**. Provide an appropriate graph. Label exactly what you have graphed. Circle the part of the graph that indicates the solution, as I have done in the notes. Does your solution match the algebraic one? (Make sure to label the intersections or x-intercepts, whichever is important to your solution.)



*I used the standard window. Notice here we are looking for where the V-shaped graph is **below** the horizontal line. This is because we are wanting where the absolute value of 2x + 3 is **less** than 8. The solution is still (-5.5, 2.5).*

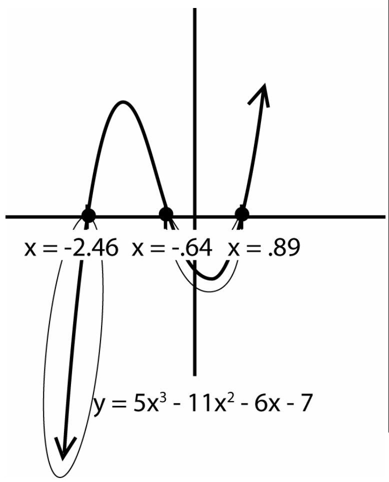
6. Polynomial and rational inequalities are dealt with in exactly the same manner as we have dealt with regular inequalities before. The only difference is that the graphs will look a little more complicated. We will start by solving the inequality $5x^3+11x^2-6x+18<25$ two different ways. Follow the instructions below.

a.) Solve $5x^3+11x^2-6x+18<25$ by graphing the left side, the right side, and seeing where the left side is below the right side. Draw and label your graph including the intersections. Also, circle the parts of the graph that correspond to the solution and use interval notation to write the solution set.



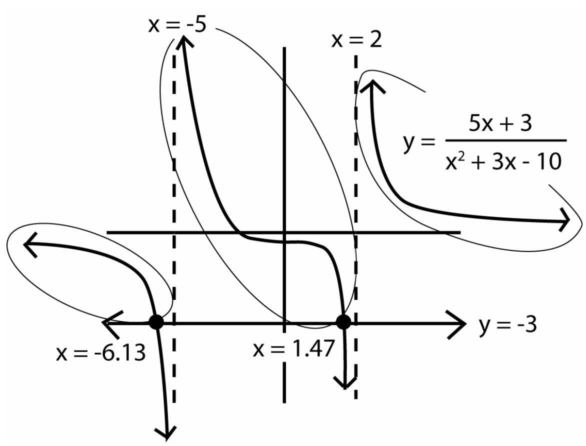
*I used the window [-10, 10] x [-10, 45]. We want where the polynomial is **below** the horizontal line. These are the two parts I have circled. In interval notation (again, focusing on the x values that give us these parts of the graph), the solution is (-∞, -2.46) and (-.64, .89). Notice we are not including any of the endpoints.*

b.) Solve $5x^3 + 11x^2 - 6x + 18 < 25$ by subtracting 25 from both sides. Then graph the left side and see where it is below the x-axis (where $y < 0$). Draw and label your graph including the x-intercepts. Also, circle the parts of the graph that correspond to the solution and use interval notation to write the solution set. Notice your answer should match exactly what you got in part a.



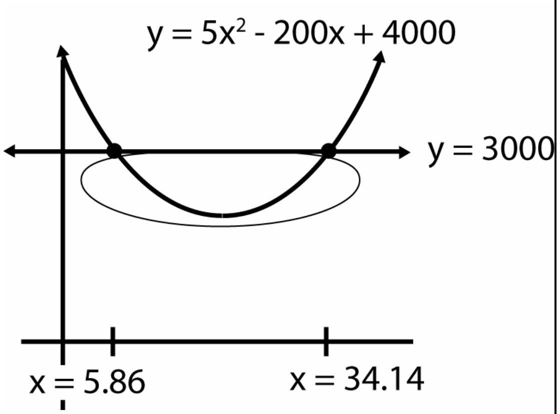
After subtracting 25 from both sides, our inequality becomes $5x^3 + 11x^2 - 6x - 7 < 0$. *So we graph* $y = 5x^3 + 11x^2 - 6x - 7$ *and see where the graph is underneath the x-axis (where y is less than zero). Notice this gives us the same answer as before. The solution is (-∞, -2.46) and (-.64, .89).*

7. Solve the inequality $\dfrac{5x+3}{x^2+3x-10} \geq -3$. Graph the left side, the right side, and see where the left side is above or equal to the right side. Draw and label your graph. (Make sure to label the intersections or x-intercepts, whichever is important to your solution.) Also, circle the parts of the graph that correspond to the solution and use interval notation to write the solution set. Remember that the expression $\dfrac{5x+3}{x^2+3x-10}$ is undefined at the values -5 and 2; make sure not to include those numbers in the solution set.



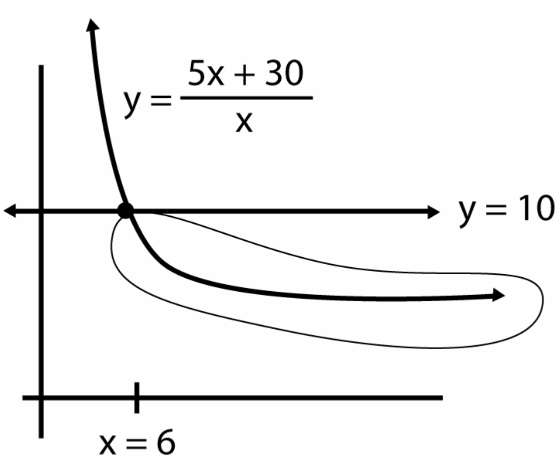
I used the standard window. The points of intersection are where we have $\dfrac{5x+3}{x^2+3x-10} = -3$ *so these x values will be a part of the solution. The parts I have circled give us* $\dfrac{5x+3}{x^2+3x-10} > -3$. *So the solution is, going from left to right, (-∞, -6.13] and (-5, 1.47] and (2, ∞). We do not include -5 or 2 because at these x values* $\dfrac{5x+3}{x^2+3x-10}$ *is undefined, not greater than or equal to -3.*

8. A manufacturer of cell phones finds that the cost $C(x)$ of manufacturing x cell phones is given by $C(x) = 5x^2 - 200x + 4000$. If the company wants to spend less than \$3000, how many cell phones can they make? Write down the inequality you need to solve. Solve this **graphically**. Draw and label your graph. (Make sure to label the intersections or x-intercepts, whichever is important to your solution.) Also, circle the parts of the graph that correspond to the solution. Write a sentence describing the solution set. (Your solution should be a range of values, telling me which x values (number of cell phones) would result in a cost of less than \$3000.)



I want to solve the inequality $5x^2 - 200x + 4000 < 3000$. *I used the window [0, 50] x [0, 5000]. I graphed the left and right sides and will see where the left side is below the right side. The intersections occur at 5.86 and 34.14. So we could say the solution is (5.86, 34.14). Since we can only make a whole number amount of cell phones, it would be correct to say (6, 34). If we want to spend less than \$3000, we need to make between 6 and 34 cell phones.*

9. An experiment on retention is conducted in a psychology class. Each student in the class is given one day to memorize 40 special characters. Every day thereafter, they are asked to write down as many of the 40 characters as they remember. The average number of characters (average of the whole class) that were remembered after x days is given by $N(x) = \frac{5x+30}{x}$ where x is greater than or equal to 1. How many days will it take for the average number of characters remembered to dip below 10? Write down the inequality you need to solve. Solve this **graphically**. Draw and label your graph. (Make sure to label the intersections or x-intercepts, whichever is important to your solution.) Also, circle the parts of the graph that correspond to the solution. Write a sentence describing the solution set.



We want to solve $\frac{5x+30}{x} < 10$. *I used a window of [0, 30] x [0, 15]. We look for where the rational function dips below the horizontal line. They intersect at x = 6. The rational function is below the horizontal line to the right of this intersection. So the solution set is (6,* ∞*). In other words, it will take 6 days for the average number of characters remembered by the whole class to dip below 10.*