Logarithmic and exponential applications Solutions NAME:
Intensity of sound

1. The loudness of a sound (that's how loud it feels to the average human ear, in decibels) is related to the ratio of the intensity of the sound to a reference level. The equation that shows this relationship is $b = 10 * \log {}^{I}\!/_{I_0}$ where ß is the loudness of the sound and I is the intensity of the sound. The I_0 is the reference level, called the hearing threshold. The hearing threshold is the very least an average person can hear. The table below gives some values of the intensity ratio ${}^{I}\!/_{I_0}$ and the corresponding loudness ß (in decibels). Notice the pattern emerging from the first several entries. Fill in the missing ${}^{I}\!/_{I_0}$ and ß values, keeping this pattern in mind. Write the ${}^{I}\!/_{I_0}$ values as 10 raised to some power.

Loudness and intensity level of some common sounds			
Source	${}^{I}\!/_{I_0}$	**ß**	**Description**
	10^0	0	Hearing threshold
Normal breathing	10^1	10	Barely audible
Rustling leaves	10^2	20	
Soft whisper (at 5 meters)	10^3	30	Very quiet
Library	10^4	40	
Quiet office	10^5	50	Quiet
Normal conversation (at 1 meter)	10^6	***60***	
Busy traffic	$\boldsymbol{10^7}$	70	
Noisy office with machines; average factory	10^8	***80***	
Heavy truck (at 15 meters); Niagara Falls	10^9	***90***	Constant exposure endangers hearing
Construction noise (at 3 meters)	$\boldsymbol{10^{11}}$	110	
Rock concert with amplifiers (at 2 meters); jet takeoff (at 60 meters)	10^{12}	***120***	Pain threshold
Pneumatic riveter; machine gun	10^{13}	***130***	
Jet takeoff (nearby)	$\boldsymbol{10^{15}}$	150	

2. If a sound has an intensity ratio I/I_0 of 6.43x10^5, what is the sound's loudness ß? Use the formula $b = 10 * \log I/I_0$. Put 6.43x10^5 in for I/I_0. Use your calculator to find ß. Also, describe a source that might have the same intensity and loudness of this sound. Circle your answer and label it as "decibels".

$$b = 10 * \log(6.43 \times 10^5)$$
$$b = 58.08$$

b = 58.08 decibels
From the table, we see this is similar to normal conversation.

> Use the LOG button. To enter the scientific notation 6.43x10^5, you need the EE function. It is the second function of the comma button (above the 7). The TI85 and TI86 have the EE function as its own button, above the 7. Review "Scientific notation and your calculator" if you do not remember how to use it.

3. The other night I measured the loudness ß of a plane flying overhead to be 125 decibels. What is the intensity ratio I/I_0 of the sound? Use the formula $b = 10 * \log I/I_0$. Put in 125 for ß. Now you must solve $125 = 10 * \log I/I_0$. This involves isolating the *log* part and then applying the inverse exponential function to undo the *log*. Remember you want to isolate I/I_0, so stop when you get there.

$$125 = 10 * \log I/I_0$$
$$12.5 = \log_{10}\left(I/I_0\right)$$
$$10^{12.5} = I/I_0$$

> *I divided both sides by 10 first. Then I rewrote the right side so it would be easier to convert to exponential form, with the base of 10 written out explicitly. Then rewrite it in exponential form. Notice this unburies the I/I_0.*

4. We measured the loudness of a rock concert 10 meters from a speaker. The loudness was measured at 117.45 decibels. What is the intensity ratio I/I_0 of the sound? Use the formula $b = 10 * \log I/I_0$. Put in 117.45 for ß. Now you must solve $117.45 = 10 * \log I/I_0$. This involves isolating the *log* part and then applying the inverse exponential function to undo the *log*.

$$117.45 = 10 * \log I/I_0$$
$$11.745 = \log_{10}\left(I/I_0\right)$$
$$10^{11.745} = I/I_0$$

> *I divided both sides by 10 first. Then I rewrote the right side so it would be easier to convert to exponential form, with the base of 10 written out explicitly. Then rewrite it in exponential form. Notice this unburies the I/I_0.*

More practice:

1. A dog barking has an intensity ratio of 3.18×10^6. Find the loudness in decibels of this dog barking.

$$b = 10 * \log\left(3.18 \times 10^6\right)$$
$$b = 65.02$$

The dog is barking at 65.02 decibels.

2. A second dog comes along and joins the first dog in barking. Now the intensity ratio of this sound is twice as much as it was with just one dog. Find the loudness in decibels of the two dogs barking together.

$$b = 10 * \log\left(2 * 3.18 \times 10^6\right)$$
$$b = 68.03$$

Calculator: $10\log(2 * 3.18\text{E}6)$

The two dogs are barking at 68.03 decibels. Notice a sound that is twice as intense is not twice as loud. Cool, huh?

3. Manuel's office building is located on a busy corner. He measures the loudness of his office building's lobby to be 68.5 decibels. How much more intense is this sound compared to the reference level I_0? (This is another way to ask for I/I_0.)

$$68.5 = 10 * \log I/I_0$$
$$6.85 = \log_{10}\left(I/I_0\right)$$
$$10^{6.85} = I/I_0$$

4. Jan is protesting against a bar in her neighborhood. She measures the sound in front of the bar for several nights. The average loudness of sounds emitting from the bar is 65 decibels. Do you think she has good support for her argument? Explain.

Look up the decibel ratings on the table on the first page. A sound that has a loudness of 65 decibels is in between normal conversation (60 dB) and busy traffic (70 dB). Jan should shut her trap because she has nothing to complain about. (Well, look at that, no math involved!)