Logarithmic / Exponential applications NAME:
Compound interest

The first part of this worksheet investigates the connection between the two formulas we will use for compound interest. We will justify for ourselves that the continuous compounding formula $A = Pe^{rt}$ works.

The second part of the worksheet focuses on solving problems involving these equations. The first four are guided examples. They should provide enough examples to get through the last four on your own.

Recall the formula for compound interest is $A = P\left(1 + {}^{r}\!/_{n}\right)^{nt}$ where *A* is the amount in an account after you invest *P* dollars (the principal) for *t* years. The interest is compounded *n* times a year at an annual interest rate *r* (decimal form).

If the account is compounded continuously, the formula is reduced to $A = Pe^{rt}$ where *A* is the amount in an account after you invest *P* dollars (the principal) for *t* years. The *r* is still the annual interest rate in decimal form. The *e* is the irrational number *e*; use the button on the left of your calculator.

1. They say that the $A = Pe^{rt}$ formula comes from the other formula when *n*, the number of times it compounds per year, is assumed to be really large or infinity.

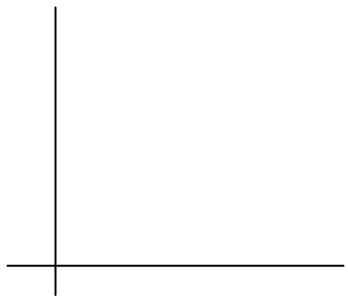
We'll look at the similarities between the two formulas. To make it easier, we'll use specific numbers for *P*, *r*, and *t* and we'll look at how the two formulas relate to each other as *n* gets bigger.

We will see that the $A = Pe^{rt}$ formula is a good estimate of $A = P\left(1 + {}^{r}\!/_{n}\right)^{nt}$ if we use really big numbers for *n*.

Copy the graphs in the given window and complete the table using the **Value** (or **Eval**) function of your calculator. (You must be in the given window.)

Suppose we invest \$100 in an account that pays interest at an annual rate of 12% for 10 years compounded *x* times a year. (Here, we're using *x* instead of *n* because our calculators require it.)

Graph both $y_1 = 100\left(1 + {.12}/{x}\right)^{(10x)}$ and $y_2 = 100e^{(.12*10)}$ using the window [0, 6] x [0, 400].



Change your window to [0, 1500] x [0, 400] and use the **Value** (or **Eval** on the TI85 or 86) function to complete the table. Enter the values for *x* at the prompt, it will tell you the *y* value for $y_1 = 100\left(1 + {.12}/{x}\right)^{(10x)}$, then arrow down and it will tell you the *y* value for $y_2 = 100e^{(.12*10)}$.

x **number of times it compounds per year**	$y = 100\left(1 + {.12}/{x}\right)^{(10x)}$	$y = 100e^{(.12*10)}$
1 once a year		
4 once every 3 months		
12 once a month		
24 once every 2 weeks		
365 once a day		
1460 once every 6 hours		

What is happening to the values of $y = 100\left(1 + {.12}/{x}\right)^{(10x)}$ in comparison to $y = 100e^{(.12*10)}$ as *x* gets bigger and bigger?

Practice using the formulas: (The previous idea is useful for understanding but not necessary to do the following problems.) Some are partially done to guide you.

1. I invest $500 in an account that pays 5% interest, compounded monthly, for a total of 15 years. How much money will the account have in it after 15 years? (Start off with the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$; put in the information that you know. Remember to write *r* in decimal form. Notice we are looking for *A*.)

Did you put 500(1 + .05/12) ^ (12*15) into your calculator? Did you get $1056.85?

2. I will invest a certain amount of money in an account that pays 15% interest compounded every two months (6 times a year). How much must I invest now to have $10,000 in ten years? (Start with the formula $A = P\left(1 + \frac{r}{n}\right)^{nt}$; put in the information that you know. Remember to write *r* in decimal form. Notice we are looking for *P*.)

Did you get $10{,}000 = P\left(1 + \frac{.15}{6}\right)^{6*10}$? Notice the complicated stuff on the right simplifies so that our equation is really $10{,}000 = P * 4.3998$. Now solve for *P*.

Did you get $2272.83 as the amount to be invested? (You would get $2272.84 if you used exact values and did not round intermediate answers.)

3. Jody invests $1000 in an account that pays 12% interest, compounded monthly. How long must she leave the money in the account in order to have $5000? (Start with the formula $A = P\left(1 + {}^{r}\!/_{n}\right)^{nt}$; put in the information that you know. Remember to write *r* in decimal form. Notice we are looking for *t*. This is a little more difficult. Remember when we solve for a variable in an exponent, we need to use the inverse relationship between the exponential and logarithmic functions.)

Did you get $5000 = 1000\left(1 + {}^{.12}\!/_{12}\right)^{12t}$? Simplify the stuff inside the parentheses and divide both sides by 1000 to get the exponential factor by itself. You should get $5 = 1.01^{12t}$. Solve it by the methods discussed in class. (I would take the natural log of both sides of the equation.)

Did you get 13.48 years?

4. Fred has $750 to invest in an account that pays 6% annual interest, compounded continuously. He wants to leave his money in the account until it has grown to $1200. How long must he wait? (Start with the formula $A = Pe^{rt}$; put in the information that you know. Remember to write *r* in decimal form. Notice we are looking for *t*. This is a little more difficult. Remember when we solve for a variable in an exponent, we need to use the inverse relationship between the exponential and logarithmic functions.)

Did you get $1200 = 750e^{.06t}$? Now divide both sides by 750 to isolate the exponential factor. Then take the natural log of both sides to unbury the exponent .06*t*. Then solve for *t* like normal.

Did you get 7.83 years?

5. Karen wants to invest $500 in an account that pays 15% annual interest, compounded monthly. How long must she leave her money in the account for it to grow to $1000?

6. Laquisha invests $600 in an account that pays 7% annual interest, compounded continuously. How much is in her account after ten years?

7. Ron needs $1000 in five years. He has decided to invest money in an account that pays 18% annual interest, compounded weekly. How much must he deposit in the account now in order to have $1000 in 5 years?

8. Harold's savings account earns 3% interest, compounded continuously. He wants to deposit $100 now. How long must he wait until the account has grown to $500?