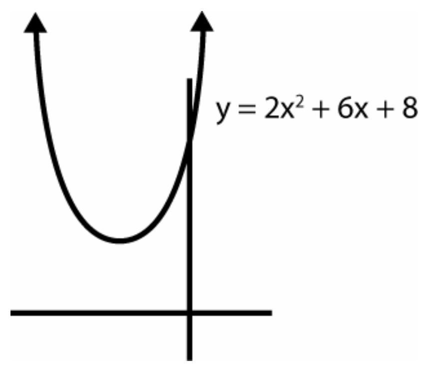
Manipulating complex numbers Solutions NAME:

This worksheet will work on understanding and manipulating complex numbers. Remember, we'll see complex numbers as solutions to quadratic equations with negative discriminants. They will be added, subtracted, and multiplied just like any binomial (an expression with two terms).

1. The equation $0 = 2x^2 + 6x + 8$ has no solution in the real numbers. We have to reach outside the real number realm to find a number that makes the equation true. Solve $0 = 2x^2 + 6x + 8$ by using your calculator's Quadratic Formula program. Draw and label the appropriate graph as well as write down the solutions to the equation. Again, the solutions are complex numbers and the graph should not touch the x-axis.



There are no real numbers that make y equal to zero. So there are no real solutions. The two complex solutions are $-1.5 \pm 1.32i$.

2. Notice that whenever there is a complex solution to an equation, the solutions come in pairs. Recall, the previous equation has the two solutions $-1.5 \pm 1.32i$. This is because of how the quadratic formula $\left(x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a} \right)$ works out. These two solutions are called conjugates. In general, the complex numbers $a + bi$ and $a - bi$ (where a and b are real numbers) are called **conjugates**. Complex solutions always appear in conjugate pairs. If $a + bi$ is a solution, then its conjugate $a - bi$ must also be a solution.

If I tell you one solution to a quadratic equation is $-6.3 + 2.5i$, then what must the other solution be?

The other solution would need to be the conjugate of $-6.3 + 2.5i$ *or* $-6.3 - 2.5i$.

3. We need to learn how to manipulate complex numbers on a basic level. They will work just like real numbers. We will use FOIL and the distribution property as we have done before. Perform the operation indicated and simplify. The first couple are done as examples.

a.) $6+4i+7-i$

$$=6+7+4i-i$$
$$=13+(4-1)i$$
$$=13+3i$$

Here, we are adding the two complex numbers 6 + 4*i* and 7 – *i*. We will use the same procedure as we would with 6 + 4*x* and 7 – *x*. Combine like terms. We can think of 4*i* – *i* as two terms with a common factor of *i*. Use the distribution property to write it as (4 – 1)*i*. Once we understand the concepts, we will not need to write so many steps down.

b.) $(3-5i)(-4+3i)$

$$=-12+9i+20i-15i^2$$
$$=-12+29i-15(-1)$$
$$=-12+29i+15$$
$$=3+29i$$

Here, we are multiplying the two complex numbers 3 – 5*i* and -4 + 3*i*. We will use the same procedure as we would with 3 – 5*x* and -4 + 3*x*. FOIL it out. The tricky part here comes with the $-15i^2$. Now, *i* is $\sqrt{-1}$. So i^2 is $\sqrt{-1}\sqrt{-1}$ which is plain old -1. Notice how we use that in the second line.

c.) $3.5-4.2i+7-2i$

Try to end up with two terms, one with *i* in it and one without.

$$=3.5-4.2i+7-2i$$
$$=3.5+7-4.2i-2i$$
$$=10.5-6.2i$$

d.) $(5+3i)(3-2i)$

$$=15-10i+9i-6i^2$$
$$=15-i-6(-1)$$
$$=15-i+6$$
$$=21-i$$

e.) $9-4i-(2+3i)$

$= 9-4i-2-3i$

$= 9-2-4i-3i$

$= 7-7i$

f.) $i^2(4+6i)+3-8i+2(4-3.6i)$

Two alternative methods for simplifying	
$=(-1)(4+6i)+3-8i+2(4-3.6i)$	$= i^2(4+6i)+3-8i+2(4-3.6i)$
$=-4-6i+3-8i+8-7.2i$	$= 4i^2+6i^3+3-8i+8-7.2i$
$=-4+3+8-6i-8i-7.2i$	$= 4(-1)+6(-i)+3-8i+8-7.2i$
$= 7-21.2i$	$=-4-6i+3-8i+8-7.2i$
	$=-4+3+8-6i-8i-7.2i$
	$= 7-21.2i$

$i^3 = i^2\, i = (-1)i = -i$

g.) $(6+3i)(6-3i)$

$= 36-18i+18i-9i^2$

$= 36-9i^2$

$= 36-9(-1)$

$= 36+9$

$= 45$

Notice, here we ended up with a plain old real number. A complex number times its conjugate will always be a real number.

h.) $4i(2-7i)+6-9i+5$

$= 8i-28i^2+6-9i+5$

$= 8i-28(-1)+6-9i+5$

$= 8i+28+6-9i+5$

$= 28+6+5+8i-9i$

$= 39-i$

i.) $(-8+6i)(3-6i)-4i+7$

$=-24+48i+18i-36i^2-4i+7$

$=-24+66i-36(-1)-4i+7$

$=-24+66i+36-4i+7$

$=-24+36+7+66i-4i$

$= 19+62i$