

Manipulating Complex Numbers 2 Solutions

NAME:

This worksheet continues working on adding, subtracting, and multiplying complex numbers. Complex numbers like $3 + 2i$ are dealt with in the same way as numbers like $3 + 2x$. We will also get practice checking complex solutions by substituting them into the original equations.

1. a.) If $i = \sqrt{-1}$, then what must i^2 be? (Hint: $i^2 = \sqrt{-1}\sqrt{-1}$)

$$\text{Just like } \sqrt{9}\sqrt{9} = 9, i^2 \text{ or } \sqrt{-1}\sqrt{-1} = -1.$$

b.) What is i^3 ? (Hint: $i^3 = i^2 * i$)

$$\text{This is } -1 \text{ times } i, \text{ or } -i.$$

c.) What is i^4 ? (Hint: $i^4 = i^2 * i^2$)

$$\text{This is } -1 \text{ times } -1, \text{ or } +1.$$

2. Simplify each of the following by performing the operation and combining like terms.

$$\begin{aligned} \text{a.) } & .35 + .65i - (.16 + .44i) \\ & = .35 + .65i - .16 - .44i \\ & = .35 - .16 + .65i - .44i \\ & = .19 + (.65 - .44)i \\ & = .19 + .21i \end{aligned}$$

Distribute the negative. Then combine like terms. Here I show how the distribution property is used to simplify the complex part.

$$\begin{aligned} \text{b.) } & 2.4i + 3i + .7i - 2.9 - .38 + 3.2 \\ & = 6.1i - .08 \\ & = -.08 + 6.1i \end{aligned}$$

Combine like terms; you have terms with i in them and terms without i . Usually we write complex numbers with the i term second so I rewrote my answer.

$$\begin{aligned} \text{c.) } & 4i^2 + 3i - 5 - (3 - 2i) \\ & = 4(-1) + 3i - 5 - 3 + 2i \\ & = -4 - 5 - 3 + 3i + 2i \\ & = -12 + 5i \end{aligned}$$

Distribute the negative at the end. At the same time, recognize i^2 as -1 . Then get like terms together.

$$\begin{aligned} \text{d.) } & (5 + .4i)(5 - .4i) \\ & = 25 + 2i - 2i - .16i^2 \\ & = 25 - .16(-1) \\ & = 25 + .16 \\ & = 25.16 \end{aligned}$$

We use FOIL to multiply these. Notice how the inside and outside terms cancel out and the i^2 is thought of as -1 . We end up with a real number when we multiply a complex number by its conjugate. There are no i 's left.

3. The following equations are given with their complex solutions. Check both solutions by substituting them into the original equation to see if they work. Some solutions are rounded.

a.) $-13 = x^2 - 6x$ Solutions: $3 \pm 2i$

$-13 \stackrel{?}{=} (3+2i)^2 - 6(3+2i)$ $-13 \stackrel{?}{=} 9 + 6i + 6i + 4i^2 - 18 - 12i$ $-13 \stackrel{?}{=} 9 - 18 + 4i^2$ $-13 \stackrel{?}{=} 9 - 18 + 4(-1)$ $-13 \stackrel{?}{=} 9 - 18 - 4$ $-13 = -13$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">It works!</div>	$-13 \stackrel{?}{=} (3-2i)^2 - 6(3-2i)$ $-13 \stackrel{?}{=} 9 - 6i - 6i + 4i^2 - 18 + 12i$ $-13 \stackrel{?}{=} 9 - 18 + 4i^2$ $-13 \stackrel{?}{=} 9 - 18 + 4(-1)$ $-13 \stackrel{?}{=} 9 - 18 - 4$ $-13 = -13$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">It works!</div>
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b.) $x^2 - .8x + .2 = 0$ Solutions: $.4 \pm .2i$

$(.4 + .2i)^2 - .8(.4 + .2i) + .2 \stackrel{?}{=} 0$ $.16 + .08i + .08i + .04i^2 - .32 - .16i + .2 \stackrel{?}{=} 0$ $.16 + .04(-1) - .32 + .2 \stackrel{?}{=} 0$ $.16 - .04 - .32 + .2 \stackrel{?}{=} 0$ $0 = 0$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">It works!</div>	$(.4 - .2i)^2 - .8(.4 - .2i) + .2 \stackrel{?}{=} 0$ $.16 - .08i - .08i + .04i^2 - .32 + .16i + .2 \stackrel{?}{=} 0$ $.16 + .04(-1) - .32 + .2 \stackrel{?}{=} 0$ $.16 - .04 - .32 + .2 \stackrel{?}{=} 0$ $0 = 0$	<div style="border: 1px solid black; padding: 5px; display: inline-block;">It works!</div>
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c.) $0 = 2x^2 + 3x + 2$ Solutions: $-.75 \pm .66i$

$0 \stackrel{?}{=} 2(-.75 + .66i)^2 + 3(-.75 + .66i) + 2$ $0 \stackrel{?}{=} 2(.5625 - .495i - .495i + .4356i^2) + 3(-.75 + .66i) + 2$ $0 \stackrel{?}{=} 1.125 - .99i - .99i + .8712i^2 - 2.25 + 1.98i + 2$ $0 \stackrel{?}{=} 1.125 - .8712 - 2.25 + 2$ $0 \stackrel{?}{=} .0038$	<div style="border: 1px solid black; padding: 10px; display: inline-block;">It works! These solutions are rounded. So when we substitute them back in, we have to expect a rounding error.</div>
$0 \stackrel{?}{=} 2(-.75 - .66i)^2 + 3(-.75 - .66i) + 2$ $0 \stackrel{?}{=} 2(.5625 + .495i + .495i + .4356i^2) + 3(-.75 - .66i) + 2$ $0 \stackrel{?}{=} 1.125 + .99i + .99i + .8712i^2 - 2.25 - 1.98i + 2$ $0 \stackrel{?}{=} 1.125 - .8712 - 2.25 + 2$ $0 \stackrel{?}{=} .0038$	<div style="border: 1px solid black; padding: 10px; display: inline-block;">It works! These solutions are rounded. So when we substitute them back in, we have to expect a rounding error.</div>