Review Exercises Solutions
College Algebra

These problems are examples of some of the prerequisite material for this class. We will spend little time on this material in class. If you are having difficulty, see me during my office hours or refer to the review material in your textbook.

1. The purpose of this question is to make sense of the rules of exponents. I find it is easier to think through the rules rather than relying on memorization.

The rules of exponents can be understood using the properties of real numbers such as the commutative and associative properties of multiplication (They are $a*b = b*a$ and $a(b*c) = (a*b)c$ for all real numbers a, b, and c.) as well as the basic ideas of exponentiation and arithmetic. For each of the rules listed below, use the given values for the variables and the properties of real numbers to help make sense of the rules. The first one is done for you.

Rule	**Specific values for variables**	**Expanded form**
$\frac{a^n}{a^m} = a^{n-m}$ where $a \neq 0$	$a = 2$, $n = 5$, $m = 3$	$\frac{2^5}{2^3} = \frac{2*2*2*2*2}{2*2*2} = \left(\frac{2*2*2}{2*2*2}\right)*\frac{2*2}{1} = 2*2 = 2^2 = 2^{5-3}$
$a^n a^m = a^{n+m}$	$a = 2$, $n = 5$, $m = 3$	$2^5 * 2^3 = (2*2*2*2*2)(2*2*2) = 2^8 = 2^{5+3}$ Use what you know about what it means to raise a number to an exponent.
$(a^n)^m = a^{nm}$	$a = 4$, $n = 2$, $m = 3$	$(4^2)^3 = (4*4)^3 = (4*4)(4*4)(4*4) = 4^6 = 4^{2*3}$ Use what you know about what it means to raise a number to an exponent.
$(ab)^n = a^n b^n$	$a = 2$, $b = 3$, $n = 3$	$(2*3)^3 = (2*3)(2*3)(2*3) = (2*2*2)(3*3*3) = 2^3 * 3^3$ Use what you know about exponents. Then use commutativity of multiplication to change the order of the multiplication. You'll see how the rule comes out.
$\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$ where $b \neq 0$	$a = 2$, $b = 3$, $n = 3$	$\left(\frac{2}{3}\right)^3 = \left(\frac{2}{3}\right)\left(\frac{2}{3}\right)\left(\frac{2}{3}\right) = \frac{2^3}{3^3}$ Use what you know about multiplying fractions. Multiply the tops and bottoms separately.

$\left(\frac{a}{b}\right)^{-n} = \left(\frac{b}{a}\right)^{n}$ where $a \neq 0$, $b \neq 0$	$a = 2$, $b = 3$, $n = 1$	$\left(\frac{2}{3}\right)^{-1} = \frac{2^{-1}}{3^{-1}} = \frac{1/2}{1/3} = \frac{1}{2} * \frac{3}{1} = \frac{3}{2} = \left(\frac{3}{2}\right)^{1}$ Use the rule from above and your knowledge about dividing fractions and negative exponents. This one is a little tougher.

2. There are two exponent rules that we will use quite a lot. I think of them as more notation than rules but they need to be memorized. They are $a^0 = 1$ and $a^{-n} = \frac{1}{a^n}$ (for any real numbers a and n, $a \neq 0$). Use these rules to simplify $\frac{7^0 * x^{-3}}{x}$.

$$\frac{7^0 * x^{-3}}{x} = \frac{1 * 1/x^3}{x} = \frac{1/x^3}{x} = \frac{1/x^3}{x/1} = \frac{1}{x^3}\frac{1}{x} = \frac{1}{x^4}$$

First, deal with the 7^0. Then deal with the x^{-3} by rewriting it as "1 over, x cubed". If you recognize the x on bottom as "x over 1", you might see the next step easier. That's why I wrote it out that way. We're dividing two fractions, so flip the bottom one and multiply. Then we're multiplying two fractions, so multiply the tops and bottoms separately.

3. Simplify the following. Write your answer with positive exponents. Remember the order of operations: parentheses, exponents, multiply and divide, add and subtract.

a.) $\frac{t^2 t^{-3}}{r^4 t} = \frac{t^2}{r^4 t t^3} = \frac{t^2}{r^4 t^4} = \frac{1 * t^2}{r^4 * t^2 * t^2} = \frac{1}{r^4 * t^2}$

Deal with the negative exponent by putting "t to the positive 3" on bottom. Then simplify the bottom by noticing that $tt^3 = t^4$. Recognize the top is "1 times, t squared" so when we cancel the t^2, we'll have that 1 left on top. Do not forget to write it.

b.) $\left(\frac{m^2 y^0}{my^2}\right)^3 = \left(\frac{m^2}{my^2}\right)^3 = \left(\frac{m^2}{my^2}\right)\left(\frac{m^2}{my^2}\right)\left(\frac{m^2}{my^2}\right) = \frac{m^6}{m^3 y^6} = \frac{m^3}{y^6}$

First, deal with y^0. It's just 1, so that factor is gone. Then, we're simply cubing this fraction, so multiply it by itself three times. When multiplying fractions, we multiply tops and bottoms separately. On top, picture six m's multiplied together; that is m^6. On bottom, you may need to picture it as $(my^2)(my^2)(my^2) = (mmm)(y^2 y^2 y^2) = m^3 y^6$. Then, you've got a common factor of "m cubed" on top and bottom, so cancel that.

c.) $\left(\frac{a+b}{b^2}\right)^{-2} = \left(\frac{b^2}{a+b}\right)^2 = \left(\frac{b^2}{a+b}\right)\left(\frac{b^2}{a+b}\right) = \frac{b^4}{(a+b)^2}$

Flip the fraction and make the negative exponent positive. This uses the last rule stated in question 1. Then, it's a matter of squaring our fraction, so write it out as multiplied by itself. Then multiply the fractions, tops and bottoms separately. You do not need to bother to FOIL out the bottom.

d.) $\left(\frac{r^4t^{-2}}{rt}\right)\left(t^2\right)^{-2} = \left(\frac{r^4}{rtt^2}\right)\left(\frac{1}{\left(t^2\right)^2}\right) = \left(\frac{r^3}{t^3}\right)\left(\frac{1}{t^4}\right) = \frac{r^3}{t^7}$

Deal with the negative exponents first. In the first fraction, the "*t* to -2, on top" becomes "*t* to positive 2, on bottom". The factor "*t* squared, raised to -2" can be rewritten as "1 over, *t* squared raised to positive 2". Then we simplify within each factor and then multiply the fractions to get the final answer.

e.) $\left(\frac{3t}{12t^3}\right)^{-1} + 7t^2 = \left(\frac{12t^3}{3t}\right)^1 + 7t^2 = \left(\frac{12t^3}{3t}\right) + 7t^2 = \left(\frac{3*4t^3}{3t}\right) + 7t^2 = 4t^2 + 7t^2 = 11t^2$

Take care of the negative exponent as we did in question 3c. Then simplify that fraction. It turns out to be just $4t^2$. Notice there are common factors of 3 and *t* on top and bottom. Then add like terms to get the final answer.

4. Factor the following.

a.) $x^2 - 6x - 7 = (\quad)(\quad) = (x \quad)(x \quad) = (x+1)(x-7)$

The worksheet "Factoring trinomials" will help you with these problems if you're having difficulty. I used Reverse FOIL to factor this one. Remember, to factor something means to write it as "something **times** something" instead of "something **plus** something **plus** something". I drew in the parentheses and I knew the first terms in the parentheses would have to multiply to make x^2, so I made them both x. Then I think of two numbers that would multiply to make -7 and add to make -6. The numbers are -7 and +1. So that gives me the factors "x plus 1" and "x minus 7". This will be explained more as we do more problems. Once you get the answer, multiply it out in your head to check. And, do not try to go further by solving for *x*; it is not proper to do here.

b.) $2t^2 - 13t + 15 = (\quad)(\quad) = (2t \quad)(t \quad) = (2t-3)(t-5)$

Again, reverse FOIL helped me out here. Check the answer in your head (or on scratch paper). The A-C method of factoring, shown on the worksheet "Factoring trinomials", is also a good way to get this one factored.

c.) $2x^2t - 2xt - 12t = 2t\left(x^2 - x - 6\right) = 2t(x \quad)(x \quad) = 2t(x-3)(x+2)$

Before using reverse FOIL, I factor out the $2t$ from all terms. This step makes factoring what is left much easier. When I'm factoring $x^2 - x - 6$, I think of two numbers that multiply to make -6 and add to make -1.

5. Perform the operation and simplify. Remember the order of operations: parentheses, exponents, multiply and divide, add and subtract.

a.) $4t^2 + 5t - 3 - (3t - 2) = 4t^2 + 5t - 3 - 3t + 2 = 4t^2 + 2t - 1$

Distribute the negative sign through (3*t* – 2). Notice this changes both signs. Then combine like terms. Do not bother to try to factor the final answer; it is not possible. The point of this problem is to practice distributing negatives and combining like terms.

b.) $(3t - 2)(t + 4) - 2t^2 + 3 = 3t^2 - 2t + 12t - 8 - 2t^2 + 3 = t^2 + 10t - 5$

FOIL out the stuff in parentheses first. Then combine like terms.

c.)

$$4(x+4)^2 - 3x^2 = 4(x+4)(x+4) - 3x^2 = 4(x^2 + 8x + 16) - 3x^2$$
$$= 4x^2 + 32x + 64 - 3x^2 = x^2 + 32x + 64$$

It's good to write out "*x*+4 times *x*+4" so you remember to FOIL it. So FOIL it out first, them multiply the 4 through. Then combine like terms.

d.)

$$5(t^2+3)^2 - 3(t-2) = 5(t^2+3)(t^2+3) - 3(t-2) = 5(t^4 + 6t^2 + 9) - 3(t-2)$$
$$= 5t^4 + 30t^2 + 45 - 3t + 6 = 5t^4 + 30t^2 - 3t + 51$$

Again, write it out so you remember to FOIL the "t^2+3, quantity squared". Then distribute the 5 through that term and the -3 through the last set of parentheses. Check yourself as you go to help keep the signs straight. Combine like terms and you're done.

6. Perform the operation and simplify. It helps to remember how to add, subtract, multiply, and divide ordinary fractions like $\frac{1}{2} + \frac{4}{5}$ or $\frac{3/2}{6/7}$; the rules are the same.

a.) $\frac{4x+2}{x+1} * \frac{5x^2}{3} = \frac{(4x+2)(5x^2)}{(x+1)*3} = \frac{(5x^2)(4x+2)}{3(x+1)}$

Multiply the tops and bottoms separately. Use parentheses to keep it all straight. I put the factors $5x^2$ and 3 out in front so they don't get lost. Remember the parentheses are needed around "4*x* + 2" and "*x* + 1". It is not necessary to multiply it all out.

b.) $\dfrac{3t^2-2}{t+3}+\dfrac{5t}{t+3}=\dfrac{3t^2-2+5t}{t+3}=\dfrac{3t^2+5t-2}{t+3}$

Just like if we were adding "$^1/_4$ + $^2/_4$", we add the tops and keep the bottom as is. This is because we have like denominators. (For "$^1/_4$ + $^2/_4$", both fractions have a denominator of 4. Our algebraic fractions both have a denominator of "x + 3".) Once you add the two tops, write it in decreasing-exponent order. It is easier to compare answers if we all try to write the answers in the same form. Later, it will be much more important you do this.

c.)

$$\frac{4r+5}{r+1}-\frac{3r^2}{2r+2}=\frac{4r+5}{r+1}-\frac{3r^2}{2(r+1)}=\frac{2}{2}\frac{(4r+5)}{(r+1)}-\frac{3r^2}{2(r+1)}=\frac{2(4r+5)}{2(r+1)}-\frac{3r^2}{2(r+1)}$$

$$=\frac{2(4r+5)-3r^2}{2(r+1)}=\frac{8r+10-3r^2}{2(r+1)}=\frac{-3r^2+8r+10}{2(r+1)}$$

Here, we are subtracting two fractions with unlike denominators. So we have to make them the same before we can subtract. Notice the first denominator is "r + 1" and the second is "2(r + 1)". If we multiply the first by 2, it will be equal to the second. But we cannot just multiply the bottom of a fraction by 2; we have to multiply both top and bottom by 2. Notice we are really just multiplying this fraction by "2 over 2" or 1. This gives us two fractions with the same denominator. Subtract the tops, distribute the 2 through "4r + 5", and write it in decreasing-exponent order.

d.) $\dfrac{6t^2-1}{t+2}\div\dfrac{3t}{2t+1}=\dfrac{6t^2-1}{t+2}*\dfrac{2t+1}{3t}=\dfrac{(6t^2-1)(2t+1)}{(t+2)(3t)}=\dfrac{(6t^2-1)(2t+1)}{(3t)(t+2)}$

To divide two fractions, you flip the bottom one and multiply them. Multiply the tops and bottoms separately. Then I pulled the 3t to the front of the bottom so it would not get lost.

7. All of the exponent rules in problem 1 hold true if the exponents are rational numbers or fractions. Use the rules to simplify the following.

a.) $\left(t^{4/5}t^{2/3}\right)^{1/3}=\left(t^{22/15}\right)^{1/3}=t^{22/45}$

Inside the parentheses, add the exponents $^4/_5$ and $^2/_3$. To do this, get a like denominator of 15. You should get "$^{12}/_{15}$ + $^{10}/_{15}$" which is $^{22}/_{15}$. Now we have something raised to a power, raised to another power. Multiply the exponents and we're done.

b.) $(8x^2)^{1/3}=8^{1/3}*(x^2)^{1/3}=2x^{2/3}$

Apply the power $^1/_3$ to all factors within the parentheses. The cube root of 8, or 8 raised to $^1/_3$ is 2. To deal with the x's, you might want to think about it as $\left(x^{\frac{2}{1}}\right)^{\frac{1}{3}}$ and then multiply the exponents.

c.) $\left(4r^3t^{1/2}\right)^{3/5} = 4^{3/5} * \left(r^3\right)^{3/5} * \left(t^{1/2}\right)^{3/5} = 4^{3/5} * r^{9/5} * t^{3/10}$

First, apply the exponent $^3/_5$ to all three factors within the parentheses. Then simplify as needed. Since $4^{3/5}$ is a decimal and not a whole number, we'll just leave it as that. The other two parts follow what we have been doing with the exponent rules

8. It happens to be true that $x^{1/2}$ is just another name for $\sqrt{x}$ and $x^{1/3}$ is another name for $\sqrt[3]{x}$. In general remember that $x^{1/n} = \sqrt[n]{x}$ where *n* is a positive integer. To investigate this further, rewrite the expressions below with rational exponents. Simplify exponents as needed.

a.) $\sqrt[3]{30} * \sqrt{10} = 30^{1/3} * 10^{1/2}$

These problems are just to introduce and practice the notation.

b.) $\left(\sqrt[3]{45}\right)^2 = \left(45^{1/3}\right)^2 = \left(45^{1/3}\right)^{\frac{2}{1}} = 45^{2/3}$

Here, rewrite the stuff inside the parentheses. Then think of 2 as "2 over 1"; this reminds us how to multiply the fractional exponents.

c.) $\sqrt[5]{42} - \frac{1}{\sqrt{35}} = 42^{1/5} - \frac{1}{35^{1/2}} = 42^{1/5} - 35^{\left(-1/2\right)}$

Again, this is just practice for the notation. The fifth root of 42 can be written as "42 raised to $^1/_5$". Likewise, rewrite the square root of 35 using a fractional exponent. The last step, which is optional, entails writing it using negative exponents.