Solving exponential and logarithmic equations Solutions NAME:

Here, we will solve exponential and logarithmic equations a few different ways to give you solid examples with which to study. Try the suggested methods.

1. Solve $\log_6(3x+4)=3$. Do so by using the equivalent forms $x=b^y$ and $y=\log_b x$. By this, I mean simply rewrite the equation in exponential form. The solution will follow shortly.

$$\log_6(3x+4)=3$$
$$6^3=3x+4$$
$$216=3x+4$$
$$212=3x$$
$$70.67=x$$

The second line comes from transforming the log equation into an exponential equation. Notice this unburies the x from within the log.

Then it's a matter of figuring 6^3 as 216, and solving for x by subtracting 4 and dividing by 3.

2. Solve $\log_6(3x+4)=3$. Do this by applying the function $y=6^x$ to both sides. This gets us $6^{\log_6(3x+4)}=6^3$. Then use your log rules to simplify the left side. The solution will follow shortly.

$$\log_6(3x+4)=3$$
$$6^{\log_6(3x+4)}=6^3$$
$$3x+4=216$$
$$3x=212$$
$$x=70.67$$

For the left hand side, to get from the second line to the third line, think of $\log_6(3x+4)$ as "the number to which I raise 6 to get 3x + 4". But then we raised 6 to this number (second line); I should get 3x + 4 (third line). Solve from there as we did before.

3. Solve $3^{x^2} = 14$. Do so by taking the natural log of both sides. Then you'll use your log rules to simplify.

$$3^{x^2} = 14$$
$$\ln\left(3^{x^2}\right) = \ln(14)$$
$$x^2 \ln(3) = \ln(14)$$
$$x^2 * 1.0986 = 2.6391$$
$$x^2 = 2.4022$$
$$x = \pm 1.55$$

On the left, use the rule $\log_a M^r = r * \log_a M$ *to get to the third line. Round intermediate answers to four decimal places. (But it's best if you can get your calculator to use the exact values.) Remember the* $\pm$ *when you square root both sides.*

4. Solve $3^{x^2} = 14$. Do so by using the equivalent forms $x = b^y$ and $y = \log_b x$. By this, I mean simply rewrite the equation in logarithmic form. Then you'll use your change-of-base formula to simplify.

$$3^{x^2} = 14$$
$$\log_3 14 = x^2$$
$$\frac{\ln(14)}{\ln(3)} = x^2$$
$$2.4022 = x^2$$
$$\pm 1.55 = x$$

For the left side, use the change of base formula. I used natural log; you could have written $\log(14)/\log(3)$ *also. Then simplify the left side and solve like before.*

5. Solve $3^{x^2} = 14$. Do so by taking the log, base 3, of both sides. Then you'll use your log rules and change-of-base formula to simplify.

$$3^{x^2} = 14$$
$$\log_3\left(3^{x^2}\right) = \log_3(14)$$
$$x^2 = \ln(14)/\ln(3)$$
$$x^2 = 2.4022$$
$$x = \pm 1.55$$

The left side can be simplified to x^2 *by various methods. I thought through it by picturing* $\log_3\left(3^{x^2}\right) = x^2 * \log_3 3 = x^2 * 1 = x^2$.

I used the change of base formula for the right side. Again, solve as before.

The point of this worksheet is to show you different methods to solve the same problem. In the future, pick whichever method suits you best. Some problems will be easier one way than another.