Rational Functions: Vertical asymptotes NAME:

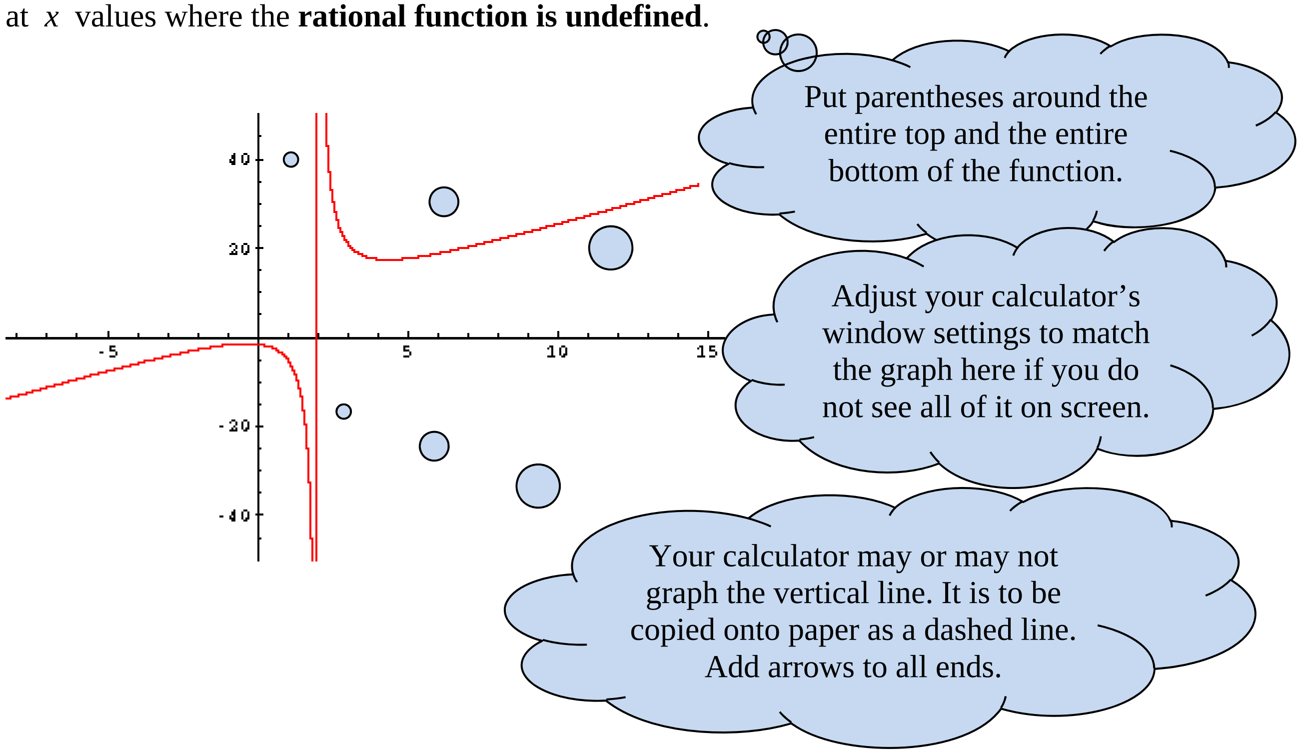
This worksheet is designed to help you understand why and where vertical asymptotes exist on the graphs of rational functions.

Recall a rational function is a function that can be written as a fraction whose numerator (top) and denominator (bottom) are both polynomial functions. You should recognize all the functions on this worksheet are indeed rational functions.

Remember a fraction is said to be “undefined” if the denominator is zero.

1. Let’s start with the function $g(x)=\dfrac{2x^2+3}{x-2}$. Algebraically find $g(2)$.

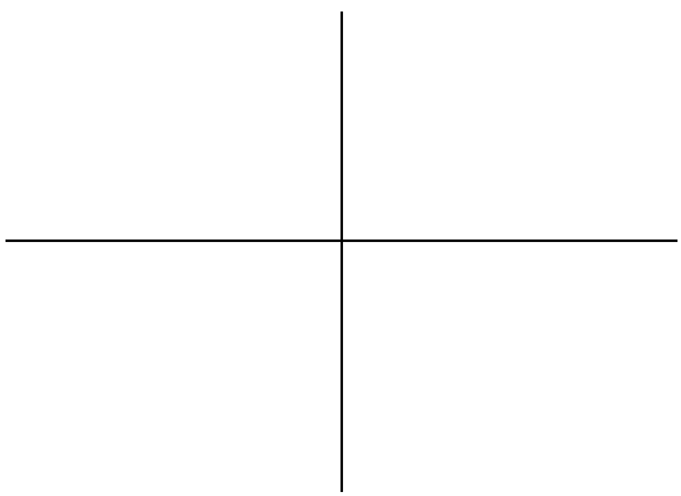
2. Using your grapher, verify that the graph of $g(x)=\dfrac{2x^2+3}{x-2}$ is shown below. Notice the vertical line at $x = 2$. This is called a **vertical asymptote**. Vertical asymptotes occur at x values where the **rational function is undefined**.



3. Where do you think the vertical asymptotes would occur on the graph of
$f(x) = \dfrac{4x+1}{(x+1)(x-3)}$?

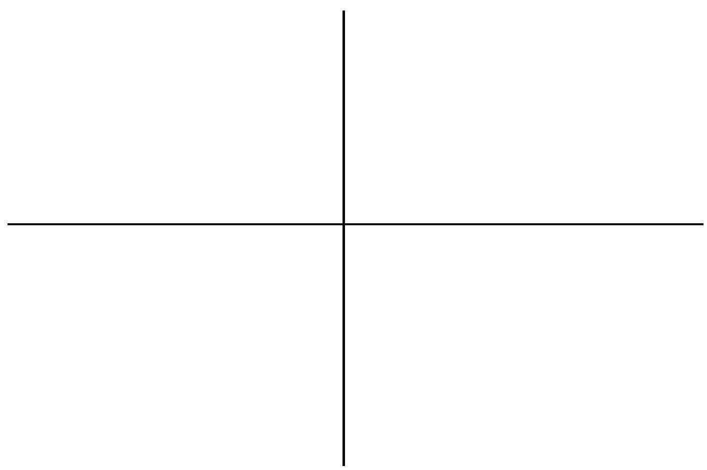
4. Graph $f(x) = \dfrac{4x+1}{(x+1)(x-3)}$ to see if you were correct. Mark and label tick marks on the *x*-axis to make your graph more accurate. **Draw the vertical asymptotes as dashed lines.**

(Note: When you enter this into your calculator, you need to make sure you have parentheses around the entire top and entire bottom; it should look like $y_1 = (4x+1)/((x+1)(x-3))$. Notice the second set of parentheses on the bottom.)



5. Where do you think the vertical asymptotes would occur on the graph of $f(x) = \dfrac{x-4}{x^2+x-6}$? (HINT: Solve $x^2 + x - 6 = 0$ to see where the denominator is zero.)

6. Graph $f(x) = \dfrac{x-4}{x^2+x-6}$ to see if you were correct. Mark and label tick marks on the *x*-axis to make your graph more accurate. (You might need to zoom in to see the part of the graph that is right of 2. I used the ZOOMIN feature.) **Draw the vertical asymptotes as dashed lines.**



7. Where do you think the vertical asymptotes would occur on the graph of $f(x) = \dfrac{2x^2-2x}{x-1}$? (Solve “bottom = 0” to see where the vertical asymptote **should** lie.)

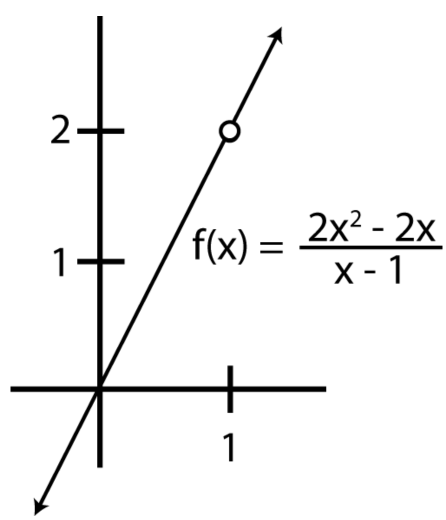
8. Graph $f(x)=\frac{2x^2-2x}{x-1}$ to see if you were correct. You will notice there is **not** a vertical asymptote at $x = 1$. Why do you think that is? Simplify $\frac{2x^2-2x}{x-1}$ to find out what's going on. (HINT: Factor the top. What can be pulled out using the distribution property?)

9. When we factor the top of $f(x)=\frac{2x^2-2x}{x-1}$, we see that we can cancel and this function reduces to simply $y=2x$. This means the function $f(x)=\frac{2x^2-2x}{x-1}$ is equivalent to the function $y=2x$ except where x is 1. Why? What happens to $f(x)$ when x is 1? Or rather, what is the $f(x)$ value that goes with the x value of 1?

So the graphs of $f(x)=\frac{2x^2-2x}{x-1}$ and $y=2x$ are identical except where x is 1.

When x is 1, the graph of $f(x)=\frac{2x^2-2x}{x-1}$ has a hole.

The graph of *f(x)* really looks like this.



The main point of this worksheet is that vertical asymptotes of rational functions are found at the x values that make the bottom zero (where the rational function is undefined). The exception to this occurs when there are common factors on the top and bottom (like (x – 1) in number 8.) In these instances, there is a hole at the x values that make the bottom zero.