

Limits and the Bouncing Ball (Section 2.2)

NAMES:

We will investigate the Flash problem “Limits and the Bouncing Ball” on my website. Go to [www.stlmath.com](http://www.stlmath.com) and click on the Flash problems link on the home page. You will find the problem listed there.

A screen shot is provided here but you should watch the animation online.

**Dropped from an initial height of 20 feet, this ball always bounces back to  $\frac{5}{6}$  of its previous height each time it bounces.**

**Assuming this continues, will the ball ever stop bouncing?**

**What if the ball had an initial height of 100 or 1,000,000 feet?**

**How many bounces would it take for the ball to bounce to a height of zero feet if the initial height is 1,000,000 feet?**

**Can you graph the relationship between the number of bounces and the height of each bounce?**

Bounce	Height (feet)
0	20.00
1	16.67
2	13.89
3	11.57
4	9.65
5	8.04
6	6.70
7	5.58
8	4.65
9	3.88
10	3.23
11	2.69
12	2.24
13	1.87
...	...

Let's work through the problems together.

1. What kind of sequence do the heights of the ball form? How do you know?

2. Complete the table for the heights of each bounce. (You will need to calculate the last two entries but you can copy the others from the animation.)

3. What is the common ratio for this sequence?

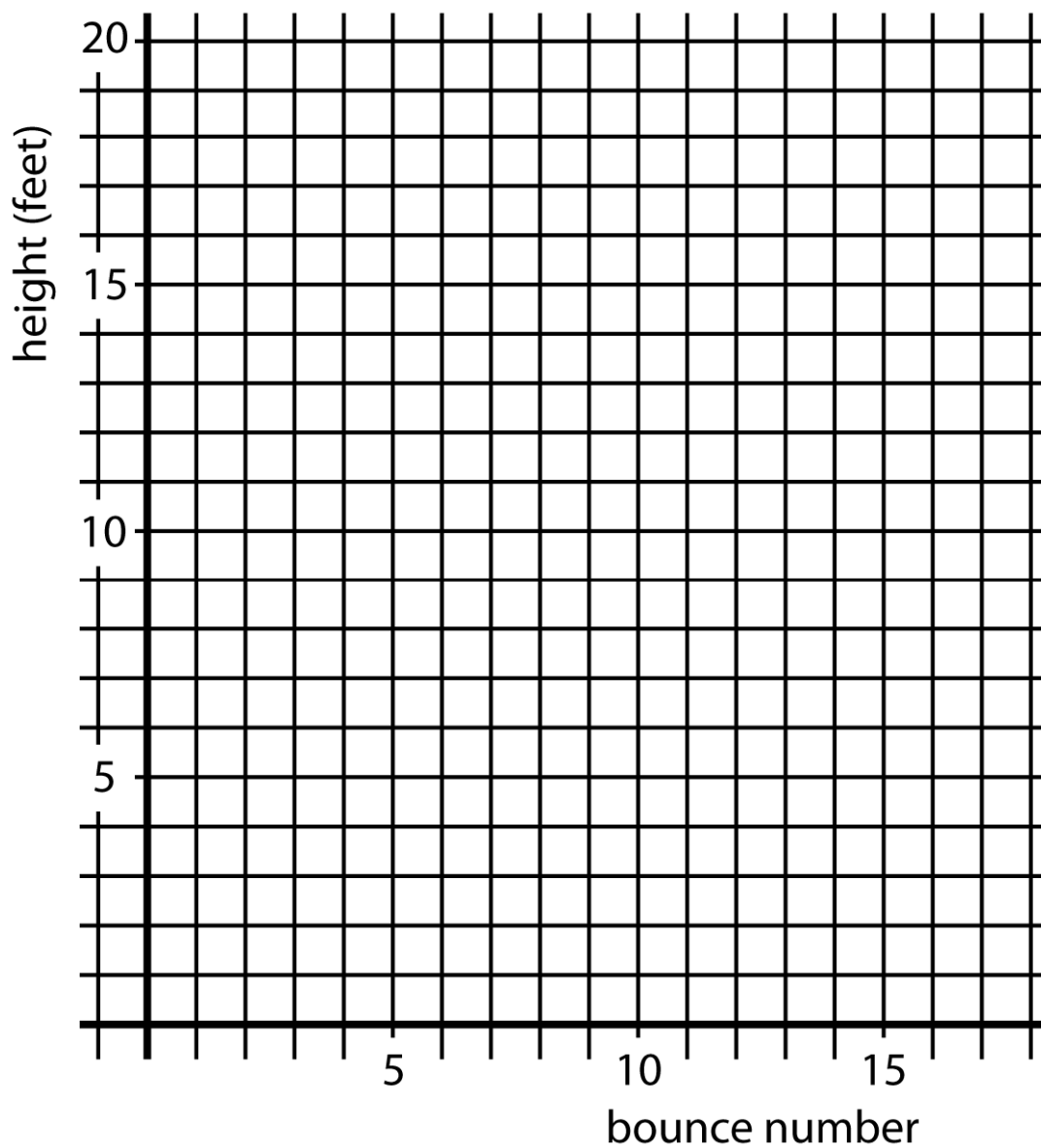
4. Do you think the ball will ever stop bouncing? (This would correspond to a height of 0 feet.) Explain.

5. The ball in the animation has an initial height of 20 feet. Assuming the same common ratio, what would be the first four terms of the sequence if the ball had an initial height of 100 feet? Round to the nearest tenth and include units.

<b>Bounce Number</b>	<b>Height of Ball (feet)</b>
0 (starting point)	20
1	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	
13	
14	
15	

6. What shape do you think the graph of this relationship takes on? Would it be a straight line, a parabola, a steady downward curve, a steady upward curve, a circle? What can you guess, before graphing, about the shape of the graph?

7. Use the graph paper below to graph the points from the table for question 2. Remember the ball has an initial height of 20 feet. Round the heights to the nearest tenths and try to graph them accurately.



8. Compare what you thought the graph would look like to what it actually looked like.

9. What is the  $x$ -intercept of the graph? (The  $x$ -intercept is where the graph hits the horizontal or  $x$ -axis. The  $y$ -value at this point is 0.) In other words, how many bounces will it take for the height to be 0 feet?

10. Conjecture as to how the graph would look if the initial height of the ball was 100 feet? Would it change shape? Would its  $x$ -intercept change? Would its  $y$ -intercept change? (The  $y$ -intercept is where the graph hits the vertical or  $y$ -axis. The  $x$ -value at this point is 0, which corresponds to 0 bounces or where the ball starts.)