Math 137 Exam 1 Review **Solutions** NAMES: **Solutions**
Chapters 1-3, 5, Inductive and Deductive Reasoning,
Fundamental Counting Principle

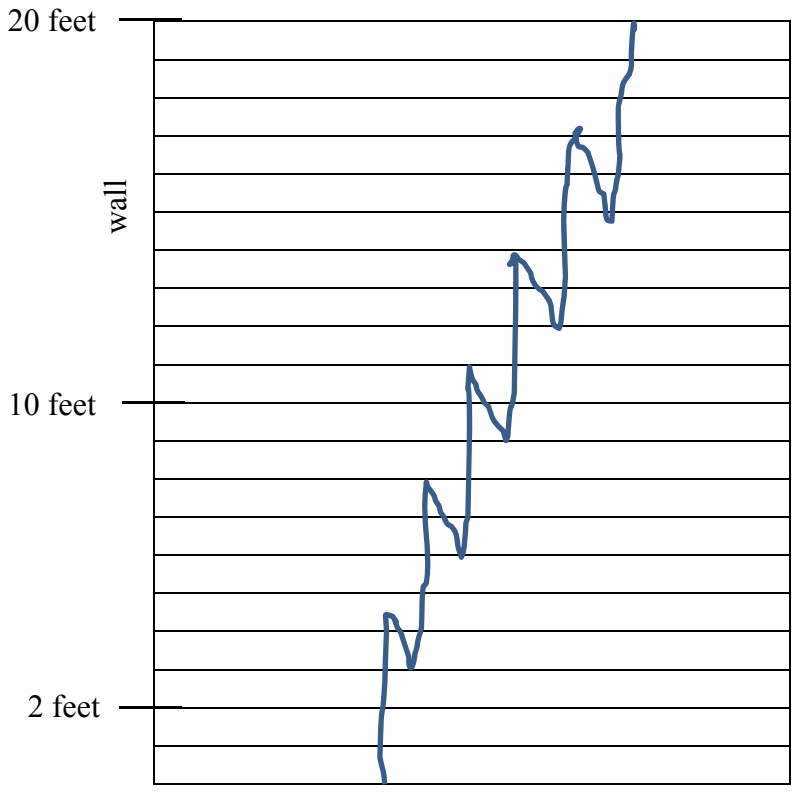
1. (3) A costume contest was held at Maria's Halloween party. Out of the 23 people who attended, prizes were given for the top three costumes. How many possible ways could the prizes be given? Explain your answer.

Solution: 23*22*21 = 10,626 possibilities

2. (4) You must draw a clear diagram for this problem. Use it to explain your solution. A blank scaled wall is provided. Label an appropriate scale and then use it to do the problem.

Amanda is learning how to rock climb. She is learning on a 20-foot climbing wall. She climbs 5 feet in 2 minutes but then slips back down 2 feet in 10 seconds. This continues until she reaches the top. How long will it take her to reach the top of the wall?

Solution: She climbs to 5 feet, and then falls to 3 feet. She climbs to 8 feet, and then falls to 6 feet. She climbs to 11 feet, and then falls to 9 feet. She climbs to 14 feet, and then falls to 12 feet. She climbs to 17 feet, and then falls to 15 feet. She climbs to 20 feet and has reached the top. Once she reaches the top, she does not fall. Total elapsed time is 2 minutes for every climb and 10 seconds for every fall, which is 12 minutes, 50 seconds.



3. (3) Make and label a systematic list of the possibilities for the rectangle described below.

A rectangle has an area of 60 square meters. If the length and width of this rectangle are whole numbers (meaning from the set {0, 1, 2, 3, …}), make a list of the possible dimensions of the rectangle. (Hint: The area of any rectangle is found by multiplying the length and width.)

Solution:

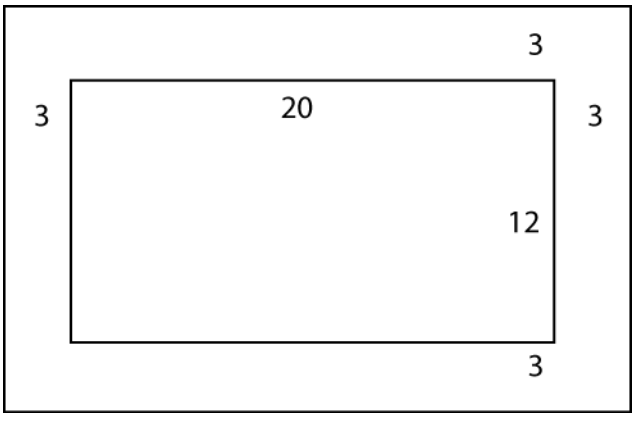
width (m)	1	2	3	4	5	6
length (m)	60	30	20	15	12	10

If you wanted to include these dimensions with the labels of width and length reversed, you could. However, you could also argue that they are essentially repeats of the rectangles listed here.

4. (3) Draw a diagram for this problem and then use it to solve the problem.

Marty is building a swimming pool. The pool measures 12 feet by 20 feet. He will build a concrete walking path directly around the pool that is 3 feet wide on all sides. What is the area of the concrete walking path? (Hint: The area of a rectangle is found by multiplying the length and width.)

Solution:



The total area (pool plus path) is 26*18 or 468 square feet. The area of the pool alone is 20*12 or 240 square feet.
We get the area of the path alone by subtracting these to get the area of the walking path is 228 square feet.

The picture helps a lot to see what we need to calculate.

5. (5) Most calculators cannot give you the exact value of $91 \times 444{,}444{,}444{,}444$ since the answer has more than ten digits. One way to find the answer is to determine if a pattern exists when 91 is multiplied by smaller numbers whose digits are all 4's. Try this method to predict the actual value of $91 \times 444{,}444{,}444{,}444$. I have included a table to get you started. Complete the table, filling in **all** the empty cells, to get full credit. (Use commas for big numbers.)

Solution:

Product	**Final Answer**
91×44	4,004
91×444	40,404
$91 \times 4{,}444$	404,404
$91 \times 44{,}444$	4,044,404
$91 \times 444{,}444$	40,444,404
91x4,444,444	404,444,404
91x44,444,444	4,044,444,404

Complete the next two entries in the first column and all entries in the second column.

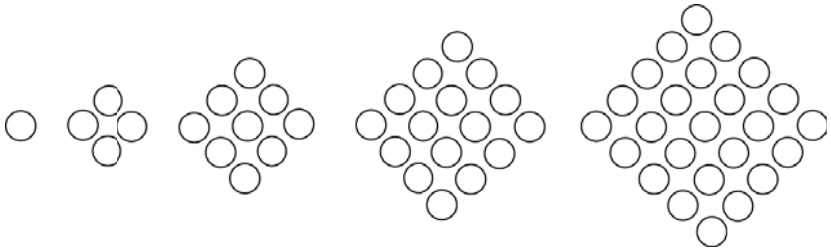
What is your prediction for the value of $91 \times 444{,}444{,}444{,}444$? (Notice it would be quite a ways further down the table.)

Solution: Each result is "40, then some 4's, then 04". Since the number of 4's in the middle of the result is two less than the number of 4's in the number we multiply 91 by, I think $91 \times 444{,}444{,}444{,}444$ will have ten 4's in the middle. So $91 \times 444{,}444{,}444{,}444$ should be 40,444,444,444,404.

6. (3) Which type of reasoning (inductive or deductive) did you use on the previous question? Explain.

Solution: I used inductive reasoning because it is based solely on seeing the pattern of many examples.

The pattern below is illustrated by these figures.



7. Complete the equations to the right.

$1 = 1$

$1+2+1 =$

$1+2+3+2+1 =$

$1+2+3+4+3+2+1 =$

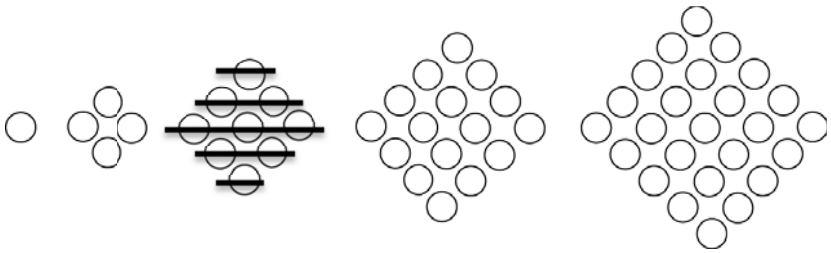
$1+2+3+4+5+4+3+2+1 =$

8. Look at the numbers on the right side of the equations. Write the sequence down and fill in the next two entries (along with the first five entries from above).

Solution: The sequence is 1, 4, 9, 16, 25. The next entries would be 36 and 49. These are the perfect squares.

9. Explain how the sums such as "1 + 2 + 3 + 2 + 1" are shown in the figures. Independently, explain how the square numbers such as "9" can be found from the figures. (This will help explain why the equations are true.)

Solution: The sum "1 + 2 + 3 + 2 + 1" can be seen by adding the circles along each horizontal row (shown below on the third picture with horizontal lines). The perfect squares can be seen because each picture is that of a number squared. For instance, the third picture (when seen turned slightly onto its side) has 3 rows of 3 circles, or 3*3 = 9 circles. The fourth picture has 4*4 or 16 circles. And so on.



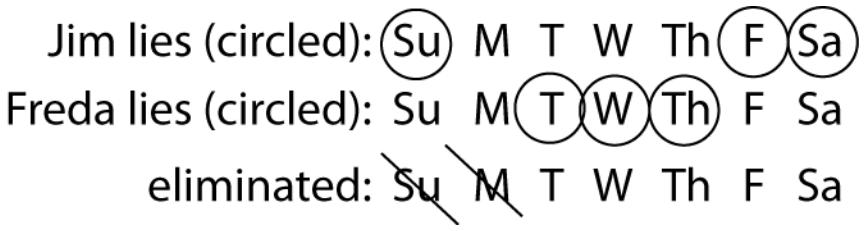
10. (3) Use the method of eliminating possibilities to solve the following problem. Give me the correct answer and then pick two other days and tell me specifically why those days cannot be the answer.

> *Jim tells lies on Fridays, Saturdays, and Sundays. He tells the truth on all other days. Freda tells lies on Tuesdays, Wednesdays, and Thursdays. She tells the truth on all other days. If they both say "Yesterday I lied," then what day is it?*

Solution: I used the chart below. It is started but not finished. I started off by assuming the day is Sunday. But that cannot be since Jim says he lied yesterday and he did lie on Saturday (which is "yesterday" if we assume it's Sunday), and that would mean he is telling the truth on Sunday but he is supposed to lie on Sundays. So it cannot be Sunday.

Next assume it's Monday. This time look at Freda. If it's Monday and she says she lied yesterday, then she is saying she lied on Sunday. Since that is untrue, she would be lying on Monday. But that cannot be since she must tell the truth on Mondays. So it cannot be Monday.

Continue like this until you can eliminate every day but Friday. It must be Friday.



11. (3) A math quiz has ten questions. They are all multiple-choice questions with four answers each. How many possible answer keys are there for this quiz? Explain.

Solution: 4*4*4*4*4*4*4*4*4*4 = 1,048,576 possible answer keys

We used the Fundamental Counting Principle and multiplied the ways that each question could be answered. You will see we have ten 4's multiplied here.

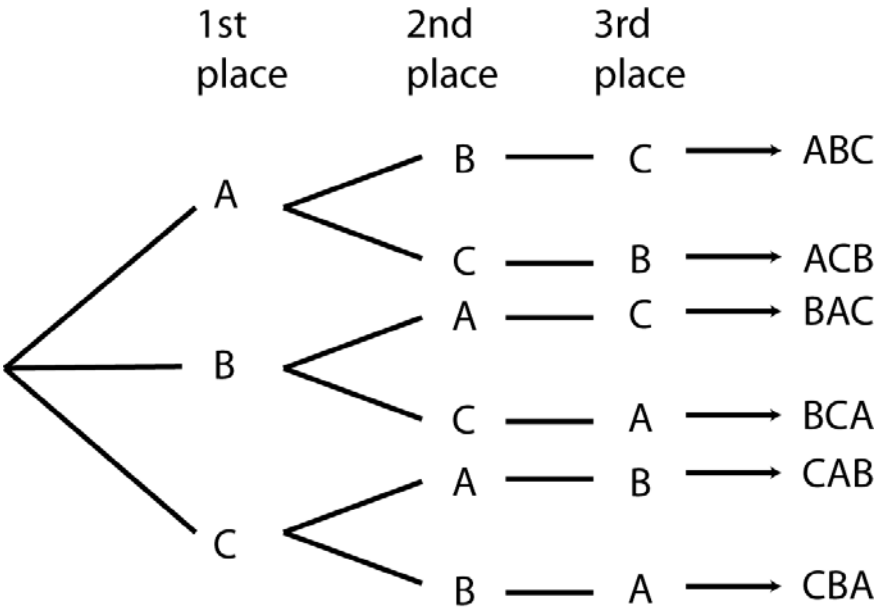
12. (3) Marty has invited ten of his friends to a movie. The first row of the theatre has five seats and he takes the seat at the end of the row (leaving four seats). How many ways can four of his ten invited friends take seats in this row?

Solution: 10*9*8*7 = 5,040 possible seating arrangements

13. (3) Draw a tree diagram to help find all of the various possibilities for this problem. Make sure your diagram has every possibility for the three places listed (preferably to the right).

Three cats named Ash, Bat, and Candy are entered in a beauty contest. How many ways can the 1st, 2nd, and 3rd places be awarded?

Solution: The tree diagram shows 6 possibilities for the top three places to be awarded.



14. (3) The Fibonacci sequence (from the pairs of rabbits problem) is reproduced below for the first few entries. Complete the next three entries and state, in words, the rule that assigns each entry.

Solution: 1, 1, 2, 3, 5, 8, 13, __21___, __34___, __55___

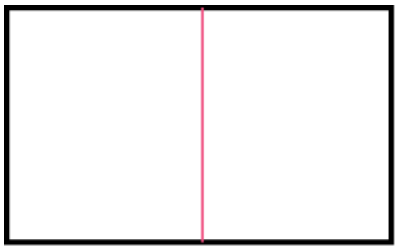
Solution: The pattern is that you add two consecutive entries to get the next entry.

15. (3) Find the 50th digit to the right of the decimal point in the decimal expansion of $\frac{7}{27}$.

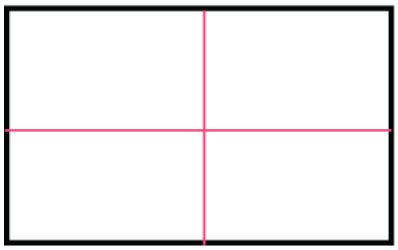
(Divide 7 by 27 on your calculator. The answer is called its decimal expansion. The answer as given will probably be rounded on the calculator so do not trust the last digit.) Explain how you arrived at your answer.

Solution: On my calculator, 7 divided by 27 was .259259259… So every third digit is 9, which means the 48th digit (since 48 is a multiple of 3) to the right of the decimal point would be 9. That would make the 49th digit a 2 (repeating the pattern) and the 50th digit would have to be a 5.

Among the terms sometimes used by the book industry to indicate the size of a book's pages are folio, quarto, and octavo. These words refer to the number of pages that can be obtained from large printer's sheets by folding them as shown by the gray or red lines in the figure on the right. Smaller pages obtained from the large sheets are referred to as 16 *mo*, 32 *mo*, and 64 *mo*.



Folio



Quarto

Octavo

16. (3) This number sequence starts off with 2, 4, 8 as shown by the pictures. Write the next three terms of this sequence.

Solution: 16, 32, 64, … The entries are merely two times the previous entry.

17. (3) Why does the folding of the sheets result in this particular sequence? Fold your sample sheet of paper if it helps.

Solution: Each time you fold the paper in half, it takes the sections and doubles them. Try it with your paper.

18. (3) Margie has many nickels. She has less than 100 nickels. When she stacks her nickels in piles of 10, there are none left over. When she stacks them in piles of 3, there are none left over. When she stacks them in piles of 4, there are none left over. How many nickels does she have? Explain.

Solution: If she puts them in piles of 10 and there are none left over, then she must have a multiple of ten. So she has either 10, 20, 30, 40, 50, 60, 70, 80, or 90 nickels.

She puts them in piles of 3 and there are none left over, so she must have a multiple of 3. That eliminates 10, 20, 40, 50, 70, and 80 from the possibilities. This leaves the only possibilities for the number of nickels are 30, 60, or 90.

When she stacks them in piles of 4, she has none left over. So we eliminate 30 and 90. That leaves one possibility for the number of nickels, 60.

19. (6) Complete each pattern. Explain each pattern in words.

Solution:

a.) 3, 6, 9, 12, __15_____, ___18____, ___21____, ___24____

In words: We add three to each number to get the next number in the list.

b.) 1000, 500, 250, 125, ___62.5____, ___31.25____, ___15.625____, ___7.8125____

In words: We halve (or divide by 2) each number to get the next number in the list.

c.) 4, 7, 14, 25, 40, ___59____, ___82____, ___109____, ___140____

In words: I noticed that from 4 to 7, we added 4. From 7 to 14, we added 7. From 14 to 25, we added 11. From 25 to 40, we added 15. It looks as though the number we add each time goes up by 4. In other words, the differences between the entries are 4, 7, 11, 15, … So I added 19 to 40 to get 59. Then I added 23 to 59 to get 82. Then I added 27 to 82 to get 109. Lastly, I added 31 to 109 to get 140.

20. (6) Do the following trick with two different numbers. Then in the fourth column of the table, write down the symbolic proof used to show that it will always work. We do this by using a little square (□) to denote the original number and little circles (∘) to denote numbers we add or subtract (like 5 or 10). Answer the question that follows the table.

Solution	First example	Second example	Symbolic Proof
step 1: Choose a number	12	7	□
step 2: Add 5	12 + 5 = 17	7 + 5 = 12	□ ∘ ∘ ∘ ∘ ∘
step 3: Multiply the result by 2	17*2 = 34	12*2 = 24	□ ∘ ∘ ∘ ∘ ∘ □ ∘ ∘ ∘ ∘ ∘
step 4: Add the original number to this result	34 + 12 = 46	24 + 7 = 31	□ ∘ ∘ ∘ ∘ ∘ □ ∘ ∘ ∘ ∘ ∘ □
step 5: Subtract 10	46 – 10 = 36	31 – 10 = 21	□ □ □

What will you always get as a result, in terms of the original number? Do more examples if you need. **Solution:** You will always get three times the original number.

21. (3) Which is your favorite problem solving method or type of problem we have learned so far. Why?

Solution: I would like you to consider each method for its advantages and disadvantages. For me, I prefer drawing diagrams sometimes but it's not useful for certain problems. I am glad to have many methods available to me.