Math 137 Exam 2 Review Solutions NAMES:
Chapters 6-10

Show your work. Be sure to include appropriate units.

1. (3) My faucet runs at a rate of 5 gallons a minute. How many quarts per hour is that? [One gallon equals four quarts.]

I used unit analysis, writing out the units and making sure they cancel to leave quarts on top and hours on bottom.

$$\frac{5 \cancel{\text{gallons}}}{1 \cancel{\text{minute}}} \bullet \frac{60 \cancel{\text{minutes}}}{1 \text{ hour}} \bullet \frac{4 \text{ quarts}}{1 \cancel{\text{gallon}}} = 1200 \text{ quarts per hour}$$

2. (10) Maggie takes three of her friends on a car trip. They travel 535 miles. Their gas cost \$2.05 per gallon. Their car has a gas mileage of about 35 miles per gallon. They drove for a total of 8 hours. Find the following.

a.) Number of gallons used

$$\frac{535\, miles}{1\, trip} \cdot \frac{1\, gallon}{35\, miles} \approx 15.286\ gallons$$

b.) Total cost of gas

$$\frac{535\, miles}{1\, trip} \cdot \frac{1\, gallon}{35\, miles} \cdot \frac{\$2.05}{1\, gallon} \approx \$31.34$$

c.) Passenger-miles

This is just the number of (non-paid) passengers times the number of miles driven.

$$4 \cdot 535 = 2{,}140\ \ passenger\, miles$$

d.) Total cost per passenger-mile

$$\frac{\$31.34}{2140} \approx \$.0146\ \ per\ \ passenger\, mile$$

e.) Average speed in miles per hour

$$\frac{535\, miles}{8\, hours} \approx 66.875\, mph$$

3. (3) Consider the trip that Maggie and her friends took in the last question. In general, it is estimated to cost about 23 cents per mile to drive a car due to wear-and-tear (not including the gas). If she and her three friends want to share this cost for the trip, how much should each friend pay Maggie (not including that friend's portion for gas)?

We find the total cost for this "wear-and-tear" to be $535\,miles \cdot \frac{23\,cents}{1\,mile} = \$123.05$.

So then, per person, we would get a cost of $\frac{\$123.05}{4\ people} \approx \$30.76$. Each friend should pay Maggie \$30.76.

4. (4) Kay wants to buy a new gas fireplace insert to replace her use of wood. She would normally burn two cords of wood in the winter. Wood costs \$260 per cord. She would like to know how much she can save by burning gas instead. Gas costs \$0.87 per 100,000 BTU. The fireplace insert would burn 30,000 BTU per hour. Kay figures she would need to turn on the fireplace for 6 hours per day for 120 days. The cost of installing the gas insert is \$120. How much will she save if she makes the switch? Show your work and include units along the way.

Cost for wood: 2 cords at \$260 each makes \$520 she would pay for one winter heated with wood

Cost for gas:

$$\frac{\$\ .87}{100{,}000\ \cancel{BTU}} \bullet \frac{30{,}000\ \cancel{BTU}}{1\ \cancel{hour}} \bullet \frac{6\ \cancel{hours}}{1\ \cancel{day}} \bullet \frac{120\ \cancel{days}}{1\ winter} = \$187.92\ per\ winter$$

That does not include the initial cost of \$120 for installing the gas fireplace insert. So the total cost for using gas for heat is \$307.92, at least for the first winter.

Subtract the cost for gas from the cost for wood to find she will save \$212.08.

5. (12) Convert from metric to English or vice versa as indicated. The only metric-to-English conversions you are allowed to use are 1 meter equals 3.281 feet and 1 gallon equals 3.79 liters. Of course, you may use any English-to-English conversions such as 1 mile equals 5,280 feet and any metric-to-metric conversions such as 1,000 meters equals 1 kilometer. Also, remember 100 centimeters (cm) is equal to 1 meter and 4 quarts make a gallon.

a.) 35 m to feet

$$\frac{35\,meters}{} \cdot \frac{3.281\ feet}{1\,meter} = 114.835\ feet$$

I write the "35 meters" this way to keep the units on top so it's easier to see how they cancel with the "meters" on bottom of the second fraction.

b.) 170 feet to meters

$$\frac{170\ feet}{} \cdot \frac{1\,meter}{3.281\ feet} \approx 51.81\,meters$$

c.) 150 mi to kilometers

$$\frac{150\,miles}{} \cdot \frac{5280\ feet}{1\,mile} \cdot \frac{1\,meter}{3.281\ feet} \cdot \frac{1\,kilometer}{1000\,meters} \approx 241.39\,kilometers$$

We are only allowed certain conversions so we cannot go directly from miles to kilometers.

d.) 4 ft to centimeters

$$\frac{4\ feet}{} \cdot \frac{1\,meter}{3.281\ feet} \cdot \frac{100\,cm}{1\,meter} \approx 121.91\,cm$$

e.) 5 gallons to liters

$$\frac{5\,gallons}{} \cdot \frac{3.79\,L}{1\,gallon} = 18.95\,L$$

f.) 16 L to quarts

$$\frac{16\,L}{} \cdot \frac{1\,gallon}{3.79\,L} \cdot \frac{4\,quarts}{1\,gallon} \approx 16.89\,quarts$$

6. (4) Your speedometer is broken. You notice it takes you 48 seconds to travel between one mile marker and the next on the highway. What is your speed in miles per hour? If the speed limit is 65, are you speeding?

The information means you went 1 mile in 48 seconds. So, let's convert that rate to miles per hour. Notice how we write the first fraction with miles on top so that the final answer will have miles on top (as in "miles per hour").

$$\frac{1 \text{ mile}}{48 \text{ seconds}} \bullet \frac{3600 \text{ seconds}}{1 \text{ hour}} = 75 \text{ miles per hour}$$

So, your car is going 75 miles per hour and you are speeding. My husband, whose speedometer was broken in an older car, tells me this is how he used to figure his speed.

7. (3) Find the sum of the numbers below.

1 + 2 + 3 + …… + 999 + 1,000

We notice that 1 + 1,000 is 1,001. Also, 2 + 999 is 1,001. Also, 3 + 998 is 1,001. This pattern continues and you can show that you can split these 1,000 numbers into 500 pairs, each having a sum of 1,001. So the sum of all of the numbers is 500 times 1,001 or 500,500.

8. (3) Use estimation to find a quick answer to the question.

The city of Amberville has 33,542 citizens and it needs to build winter weather shelters. It wants to emulate a similar city to estimate the number of cots it needs. Their sister city has 52,678 citizens and has gotten by with 2,000 cots in its emergency shelters. How many cots should the city of Amberville provide? Show your work and explain your answer.

Your answer may vary since you will estimate the numbers as you see fit.

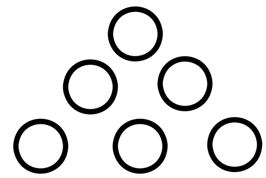
Estimate the population of the sister city at 53,000. Divide 2,000 cots by 53,000 people to get they have about .04 cots per person.

Estimate the population of Amberville at 33,500 people. Multiply the .04 by this estimate and we get Amberville needs about 1,340 cots.

9. (4) Use physical manipulatives to solve this problem. I suggest ripping up six little pieces of paper, labeling them with the digits 1, 2, 3, 4, 5, and 6. Then you can easily rearrange the pieces to come up with your answer. [There is more than one correct answer, sort of.]

Here is a magic triangle. The sum of the digits forming each side of the triangle is 11. Use the digits 1, 2, 3, 4, 5, and 6 (once each) and find the proper locations for each.

I used six labeled pieces of paper and started arranging them. I started by placing one number at the top of the triangle. What follows is my reasoning.



What happens when I place…

1 at top: Then we need a sum of 10 from the remaining numbers 2, 3, 4, 5, and 6, two different ways (for the left and right sides of the triangle to have a sum of 11). That is not possible so 1 cannot go on top.

2 at top: Then we need a sum of 9 from the remaining numbers 1, 3, 4, 5, and 6, two different ways (for the left and right sides of the triangle to have a sum of 11). We can do this (3 + 6 and 4 + 5). If we arrange those numbers down the sides of the triangle, and the 1 in the left-over spot, we see a solution. You may need to play with the numbers a bit to be sure all sides sum to 11.

3 at top: Then we need a sum of 8 from the remaining numbers 1, 2, 4, 5, and 6, two different ways (for the left and right sides of the triangle to have a sum of 11). That is not possible so 3 cannot go on top.

Continue like this. You will find two other configurations that are technically different from (but have a nice similarity to) the one found above.

10. (3) Use guess-and-check to answer the following question. Include a well-labeled table with a clear explanation as to what you did.
I accompanied a group of students to a play. Of the total number of students, 60% of them rode on a bus that had space for 75 students. If the bus was full, how many total students went to the play?

I think, "The total number of students is something we do not know. But 60% of that number is 75." So I used the first column in the table for the total number of students who went to the play.

Total number of students who attended play	**60% of that (students on the bus)**	**Is it 75?**
100	.6*100 = 60	too low
120	.6*120 = 72	too low (but close)
130	.6*130 = 78	too high
125	.6*125 = 75	presto!

The number of students that attended the play is 125.

11. (3) Use guess-and-check to answer the following question. Include a well-labeled table with a clear explanation as to what you did.
A taxi will charge me $2.35 plus $0.45 per quarter-mile. If I have $10 and want to give the driver a $2 tip, how far (to the nearest quarter-mile) can I go? [You may have some leftover change.]

I think, "I have $8 to spend for the cab fare ($10 minus tip). The fare will be $2.35 just to get in the cab. So subtract that to get $5.65. This is the amount of money we have to spend on mileage." I do not know how far I can travel, so that is column 1 in the table.

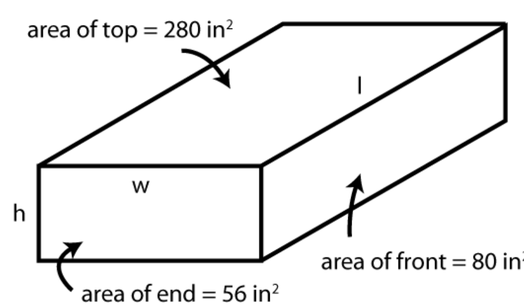
Number of quarter-miles I can travel	**Cost for quarter-miles ($)**	**Is it $5.65?**
40	.45 * 40 = 18	way too high
10	.45 * 10 = 4.50	too low
15	.45 * 15 = 6.75	too high
12	.45 * 12 = 5.40	slightly too low

Once we get that 12 quarter-miles costs us $5.40 in mileage charges, we stop. If we go an additional quarter-mile, it will cost another $.45 and our cost will be more than $5.65.

So we conclude that we can go 12 quarter-miles (or 3 miles).

12. (4) List the **sub-problems** needed to answer the following problem. Then answer the problem. You do not need to use a particular method to solve the problem.
I have drawn a rectangular box with its width, height, and length labeled for consistency among solutions. Notice the end of this box has an area of 56 square inches (width times height). The top of this box is 280 square inches (width times length). The front of this box is 80 square inches (height times length). Assume all measurements are whole numbers. What is the volume of the box? [Remember that volume is length times width times height.] A picture of this box is provided.

sub-problems:
1. find possible *h* and *w* values
2. find possible *h* and *l* values
3. find length, width, and height of box
4. find volume



Since the area of the end is 56 square inches, then the possibilities for *h* and *w* are (given as pairs *h*, *w*) 1, 56 or 2, 28 or 4, 14 or 7, 8 and those pairs reversed: 56, 1 or 28, 2 or 14, 4 or 8, 7.
(Solution continues…)

Now move on to another side. The area of the front is 80 square inches (*h* times *l*) This means *h* could not be 7. Cross off that option above. Since the other possibilities are still viable, I go on the other side.

We will work with the fact that the area of the top is 280 square inches. If we take $h = 1$ and $w = 56$, we would get that *l* must be 5 inches (because $wl = 280$). But that would make the front 5 square inches (area = *hl*) and not the 80 square inches we know it is. So we can eliminate the 1, 56 possibility for *h*, *w*. Continue like this, eliminating possible pairs until you get the only possibility left is $h = 4$, $w = 14$, and $l = 20$.

The volume is found by multiplying those dimensions, to get 1120 cubic inches.

13. (3) Use guess-and-check to answer the following question. Include a well-labeled table with a clear explanation as to what you did.
Joe skated 5 hours at an unknown rate. Marie took the same route (so skated an equal distance) but traveled for 3 hours at 4 miles per hour faster than Joe. Find both Joe's and Marie's rates. [Hint: Distance equals rate times time. Normal skating speeds range from 5 mph to about 20 mph.]

Joe's rate (mph)	**Joe's distance (miles)**	**Marie's rate (mph)**	**Marie's distance (miles)**	**Rating (Are their distances equal?)**
10	10*5 = 50	10 + 4 = 14	14*3 = 42	no
5	5*5 = 25	5 + 4 = 9	9*3 = 27	no but close
6	6*5 = 30	6 + 4 = 10	10*3 = 30	presto!

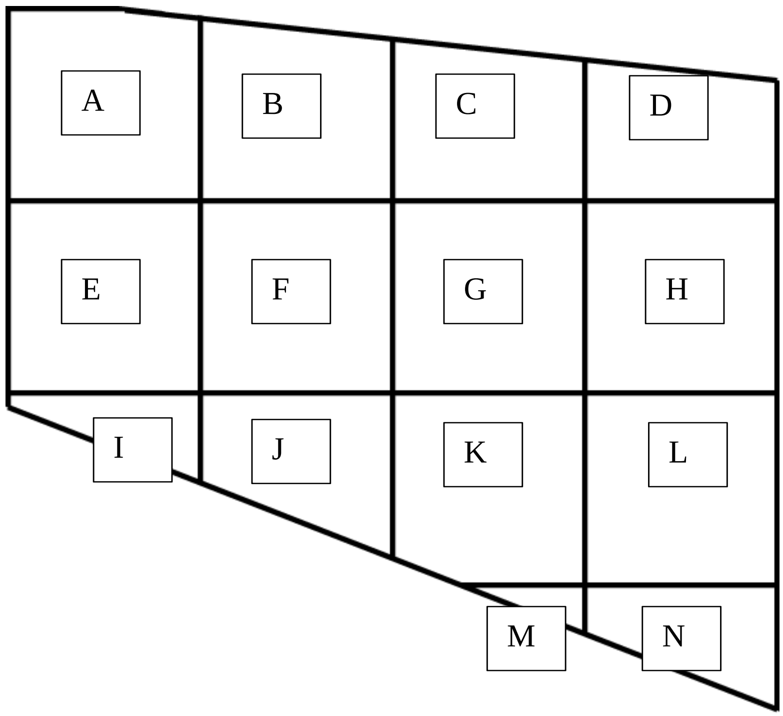
I started by using column 1 for Joe's rate since I could figure the distance he traveled by our old friend $d = r \cdot t$. I could also find Marie's rate by simply adding 4 to whatever I guessed for Joe's rate. I then used $d = r \cdot t$ again to find the distance Marie traveled. If the two distances are equal, then we guessed Joe's rate correctly.

Joe skates at 6 miles per hour. Marie skates at 10 miles per hour.

14. (3) My faucet drips one drop every two seconds. How many gallons of water does this leak waste in a year? [One quart of water is equal to about 18,927 drops. One gallon is equal to four quarts.]

$$\frac{1\ \cancel{\text{drop}}}{2\ \cancel{\text{seconds}}} \cdot \frac{1\ \cancel{\text{quart}}}{18{,}927\ \cancel{\text{drops}}} \cdot \frac{1\ \text{gallon}}{4\ \cancel{\text{quarts}}} \cdot \frac{3600\ \cancel{\text{seconds}}}{1\ \cancel{\text{hour}}} \cdot \frac{24\ \cancel{\text{hours}}}{1\ \cancel{\text{day}}} \cdot \frac{365\ \cancel{\text{days}}}{1\ \text{year}} \approx 208.27\ \text{gallons per year}$$

15. (3) Estimate the area of the figure below. The grid lines are spaced every 1 inch. Explain your answer.



I labeled the pieces with letters to help this explanation. To count them, I started off by counting the squares that were complete or mostly complete. In my mind, these are pieces A, E, F, G, H, K, and L. These were each 1 square inch, making 7 square inches.

Then I rearranged the remaining pieces in my mind. I imagined I and J coming together to make 1 square inch.

I imagined the M and N moving up and completing C and D. So that made another 2 square inches.

The only one I left out is B which is somewhat less than 1 square inch. So let's call it 75% of a whole square.

Adding up all those pieces, I get about 10.75 square inches. Your estimate might be different but it should be somewhere close.

16. (3) Solve the mixture problem below. You may use any method you choose but a guess-and-check table is started for you. If you choose guess-and-check, complete the table or develop your own. If you make your own table, please label it explicitly. **Write your answer in a complete sentence with the appropriate units.**

Becca has 20 gallons of 40% salt solution. How much 70% salt solution should she add to make a 60% salt solution?
[Hint: In the final mixture, there will be an ***unchanging*** *8 gallons of pure salt from the 20 gallons of 40% solution. Find that amount within column 3 below. There will also be an* ***unchanging*** *20 gallons of "stuff" (water and salt) from that 40% solution. Find that amount within column 4 below. These values* ***remain constant*** *through the table.]*

Number of gallons of 70% solution added	**Amount of salt coming from 70% solution (gallons)**	**Amount of salt in final mixture (gallons)**	**Total amount of stuff (water and salt) in final mixture (gallons)**	**Percentage of final mixture (decimal form)**	**Rating**
20	.7*20 = 14	8 + 14 = 22	20 + 20 = 40	22 / 40 = .55	too low
30	.7*30 = 21	8 + 21 = 29	20 + 30 = 50	29/50 = .58	too low
50	.7*50 = 35	8 + 35 = 43	20 + 50 = 70	43/70 ≈ .614	too high
40	.7*40 = 28	8 + 28 = 36	20 + 40 = 60	36/60 = .60	presto!

I first tried 30 gallons of 70% solution, but the final mixture would have had too low a concentration. So I knew I needed to add more 70% solution, and tried 50 gallons. This produced too high of a concentration for the final mixture. I tried 40 (right in the middle of 30 and 50), and it worked out to produce a final mixture of 60%.

So we should add 40 gallons of 70% salt solution to the 40% solution we already have to get a final mixture of 60%.