Math 137 Exam 3 Review Solutions
Chapters 11, 13

1. (3) I am thinking of a number. I multiplied my number by 2 , subtracted 8 , doubled the result, and added 11. Then I subtracted 6 . Then I divided by 5 . After all that, I was left with 9 . What number did I start with? Show your number is right by working the steps forwards. Please circle your answer and label the check.

Step 1: Multiply a number by 2
Step 2: Subtract 8
Step 3: Double (multiply by 2)
Step 4: Add 11
Step 5: Subtract 6
Step 6: Divide by 5
End : Ended up with 9

We undo these steps, starting with the 9 at the end. Undo step 6 by multiplying the 9 by 5 , to get 45 . Undo step 5 by adding 6 to get 51 . Undo step 4 by subtracting 11 to get 40 . Undo step 3 by dividing by 2 to get 20. Undo step 2 by adding 8 to get 28 . Undo step 1 by dividing by 2 to get 14 . So the number she started with is 14 .

The check is left for you.
2. (4) I made a lot of cupcakes for my son's bake sale. My daughter ran through the kitchen and knocked 20 of them off the table, which then needed to be thrown away. My husband came in and decided to hoard 15 cupcakes and put them in the freezer for later. My next door neighbor came by and asked for $1 / 4$ of what I had left so that she could contribute to the bake sale. So I gave her those. After that, I realized that my daughter needed to donate 40 cupcakes for her school's bake sale so I set those aside. When all was done, I had 29 cupcakes to give my son for his bake sale. How many cupcakes can I tell my son that I originally made for him? Show your number is right by working the steps forwards. Please circle your answer and label the check.

I underlined the four steps and the end result above. To work backward, start with 29 cupcakes (she had at the end) and undo the 40 cupcakes for the daughter, by adding those on to get 69 . To undo the step where $1 / 4$ of the cupcakes are given away, we need to figure that the 69 she had was the $3 / 4$ of the cupcakes that were left after the $1 / 4$ was given away. This means that each "quarter" of this amount is $1 / 3$ of 69 or 23 . That means the neighbor got 23 cupcakes. Add those to our 69 and you'll see she has 92 cupcakes right before the neighbor comes over. Then undo the 15 cupcakes hoarded in the freezer by adding those on to get 107. Lastly, undo the 20 that got knocked off the table by adding it on, to get 127. She started with 127 cupcakes.

Check: Start with 127 cupcakes, and subtract 20 that got knocked off the table, to get 107. Then subtract 15 that the husband hoards, to get 92 . The neighbor takes $1 / 4$ of those (or $92 / 4=23$ ). So subtract those off and we get 69 . Subtract the last 40 for the daughter and you'll be left with 29 cupcakes.
3. (4) My two sisters pooled their savings and decided to go on a shopping spree. They started at Macy's and each bought a pairs of jeans and a shirt, spending $\$ 45$ each. Next they went to a shoe store and one bought a pair of shoes for $\$ 28$ while the other spent $\$ 35$ on a pair. The older sister then spent $\$ 27$ on makeup. They were hungry and so spent $1 / 5$ of what they had left on lunch. They then stopped by the music store and bought 4 CDs for $\$ 9$ each. They spent half of what they had left on a gift for mom. On the way home, they spent $\$ 35$ on gas. When they got home, they had $\$ 87$. How much money did they start with?

Step 1: Macy’s: \$90
Step 2: Shoes: \$63
Step 3: Makeup: $\$ 27$
Step 4: Lunch: $1 / 5$ of what they had
Step 5: CDs: \$36
Step 6: Gift: $1 / 2$ of what they had
Step 7: Gas: \$35
End: Money left over: \$87

We know they spent $1 / 5$ of what they had on lunch. So draw a rectangle with 5 equal parts. Set aside one to represent the lunch. The other parts must equal $\$ 280$, the amount they had after lunch. So each piece is $\$ 280 / 4$ or $\$ 70$. That means lunch was also \$70.

Working backwards, we start with $\$ 87$ and add $\$ 35$ to get $\$ 122$. To undo step 6, we think that they spent half of what they had and ended up with $\$ 122$. That means they must have had twice that $\$ 122$ right before step 6. So double $\$ 122$ to get $\$ 244$. Undo step 5 by adding $\$ 36$ to this to get $\$ 280$. I drew a picture below to illustrate how we figure the cost of lunch to be $\$ 70$. Add that on and we get to $\$ 350$. Add the \$27, \$63, and \$90 for the first three steps and you get a grand total of $\$ 530$ with which they started.


4a. (3) Use guess-and-check to explore the following question. You do not need to use the table to solve the problem completely because you will be asked to solve it algebraically. The total cost of a basketball was $\$ 19.29$, including a $7.25 \%$ sales tax. How much of that cost was the price of the basketball and how much was the tax?

| Price of ball (before tax) | Tax (7.25\% of price) | Total cost (price plus tax) |
| :---: | :---: | :---: |
| $\$ 19$ | $.0725^{*} 19=1.38$ | $\$ 20.38$ too high |
| $\$ 18$ | $.0725^{*} 18=1.31$ | $\$ 19.31$ too high |
| $\$ 17.90$ | $.0725^{*} 17.90=1.30$ | $\$ 19.20$ too low |

I did not quite get to an answer but I do see that the total cost of the ball is always the price plus the tax (which is $7.25 \%$ of the price). That helps form an algebraic equation for the next step.

4b. (3) Use your guess-and-check table from part $a$ to now set up and solve an algebraic equation to answer the question. Be sure to explicitly define your variable. Answer the question with a sentence or phrase.

We know, "price of ball" + "tax" = "total cost".
Let $x=$ the price of the ball before tax
So...

$$
\begin{array}{ll}
x+.0725 x=19.29 & \begin{array}{l}
\text { We use the verbal model "price of ball" }+ \text { "tax" }=\text { "total } \\
1.0725 x=19.29
\end{array} \\
\begin{aligned}
\frac{1.0725 x}{1.0725}=\frac{19.29}{1.0725} & \text { Solve for } x \text { and you'll get the price of the ball before tax } \\
x \approx 17.99 & \text { is about } \$ 17.99 .
\end{aligned}
\end{array}
$$

5a. (3) Use guess-and-check to explore the following question. You do not need to use the table to solve the problem completely because you will be asked to solve it algebraically.
Barbie has $\$ 4.35$ in quarters and nickels. She has three more nickels than she has quarters. How many of each coin does she have?

| Number of <br> quarters | Number of <br> nickels | Total amount of money <br> (\$) |
| :---: | :---: | :---: |
| 10 | 13 | $10(.25)+13(.05)=\$ 3.15$ <br> too low |
| 15 | 18 | $15(.25)+18(.05)=\$ 4.40$ <br> too high |

I know the number of nickels is "3 more than the number of quarters". So I guessed a number of quarters and added 3 to get the appropriate number of nickels. To find the total amount of money (third column), I multiplied .25 times the number of quarters and .05 times the number of nickels, and added those.

5b. (3) Use your guess-and-check table from part $a$ to now set up and solve an algebraic equation to answer the question. Be sure to explicitly define your variable. Answer the question with a sentence or phrase.

We know, "amount in quarters" + "amount in nickels" = "total amount of money".
Let $q$ equal the number of quarters. Then, if there are three more nickels than quarters, $q+3$ must represent the number of nickels.

The equation and its solution are on the next page.

Our equation is to the right. I solved it to find the number of quarters to be 14 . So then there must be 17 nickels.

$$
\begin{aligned}
& .25 q+.05(q+3)=4.35 \\
& .25 q+.05 q+.15=4.35 \\
& .30 q+.15=4.35 \\
& .30 q=4.20 \\
& q=14
\end{aligned}
$$

6. (5) Solve the following equation. Circle your answer for $x$. Then write a problem that involves nickels and dimes that would be solved by using this equation. Proofread your problem please.

$$
.05 x+.10(2 x+1)=2.60
$$

I solved this by distributing the .10 and then combining like terms. From the third line shown here, I subtracted .10 from both sides to isolate the term with $x$. I then divided by .25 to get $x$ alone. I found $x$ to be 10 .

$$
\begin{aligned}
.05 x+.10(2 x+1) & =2.60 \\
.05 x+.2 x+.10 & =2.60 \\
.25 x+.10 & =2.60 \\
.25 x & =2.50 \\
x & =10
\end{aligned}
$$

My problem: Savvy has a pile of nickels and dimes that total $\$ 2.60$. If she has one more dime than twice the number of nickels, how many of each coin does she have?
7. (4) Below is a system of equations that could be used to solve the following problem. Solve the system and answer the question in sentence form.
Margie and Tucker went apple picking. Margie carried bags that held 6 apples each. Tucker carried bags that held 9 apples each. Together, the children picked a total of 132 apples. If Margie had two more bags than Tucker had, how many bags did each of them have? (Assume each bag was filled to capacity.)
(In the equations, $m$ represents the number of bags Margie had and $t$ represents the number of bags Tucker had.)

$$
\begin{aligned}
& m=t+2 \\
& 6 m+9 t=132
\end{aligned}
$$

I solved this by using the "substitution method for solving systems of linear equations". You can look this up on the internet. Since the first equation was already solved for $m$, I know that $m$ is the same as " $t+2$ ". So we substitute that into the second equation in place of the $m$, which gets us to the second line shown here. Distribute, combine like terms and isolate $t$ to see that Tucker had 8 bags. Since Margie had 2 more, she had 10 bags.

$$
\begin{aligned}
6 m+9 t & =132 \\
6(t+2)+9 t & =132 \\
6 t+12+9 t & =132 \\
15 t+12 & =132 \\
15 t & =120 \\
t & =8
\end{aligned}
$$

