Math 137 Exam 3 Review Solutions NAME: KEY
Chapters 11, 13, 14, 17

1. (3) I am thinking of a number. I multiplied my number by 2, subtracted 8, doubled the result, and added 11. Then I subtracted 6. Then I divided by 5. After all that, I was left with 9. What number did I start with? Show your number is right by working the steps forwards. Please circle your answer and label the check.

Step 1: Multiply a number by 2
Step 2: Subtract 8
Step 3: Double (multiply by 2)
Step 4: Add 11
Step 5: Subtract 6
Step 6: Divide by 5
End : Ended up with 9

We undo these steps, starting with the 9 at the end. Undo step 6 by multiplying the 9 by 5, to get 45. Undo step 5 by adding 6 to get 51. Undo step 4 by subtracting 11 to get 40. Undo step 3 by dividing by 2 to get 20. Undo step 2 by adding 8 to get 28. Undo step 1 by dividing by 2 to get 14. So the number she started with is 14.

The check is left for you.

2. (4) I made a lot of cupcakes for my son's bake sale. My daughter ran through the kitchen and knocked 20 of them off the table, which then needed to be thrown away. My husband came in and decided to hoard 15 cupcakes and put them in the freezer for later. My next door neighbor came by and asked for ¼ of what I had left so that she could contribute to the bake sale. So I gave her those. After that, I realized that my daughter needed to donate 40 cupcakes for her school's bake sale so I set those aside. When all was done, I had 29 cupcakes to give my son for his bake sale. How many cupcakes can I tell my son that I originally made for him? Show your number is right by working the steps forwards. Please circle your answer and label the check.

I underlined the four steps and the end result above. To work backward, start with 29 cupcakes (she had at the end) and undo the 40 cupcakes for the daughter, by adding those on to get 69. To undo the step where ¼ of the cupcakes are given away, we need to figure that the 69 she had was the ¾ of the cupcakes that were left after the ¼ was given away. This means that each "quarter" of this amount is 1/3 of 69 or 23. That means the neighbor got 23 cupcakes. Add those to our 69 and you'll see she has 92 cupcakes right before the neighbor comes over. Then undo the 15 cupcakes hoarded in the freezer by adding those on to get 107. Lastly, undo the 20 that got knocked off the table by adding it on, to get 127. She started with 127 cupcakes.

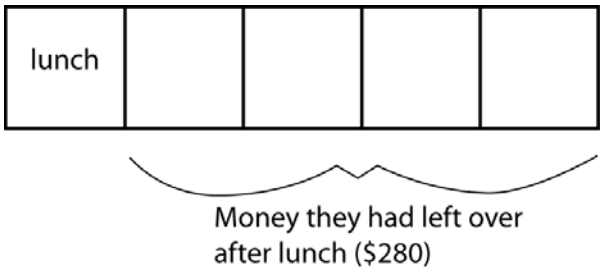
Check: Start with 127 cupcakes, and subtract 20 that got knocked off the table, to get 107. Then subtract 15 that the husband hoards, to get 92. The neighbor takes ¼ of those (or 92/4 = 23). So subtract those off and we get 69. Subtract the last 40 for the daughter and you'll be left with 29 cupcakes.

3. (4) My two sisters pooled their savings and decided to go on a shopping spree. They started at Macy's and each bought a pairs of jeans and a shirt, spending $45 each. Next they went to a shoe store and one bought a pair of shoes for $28 while the other spent $35 on a pair. The older sister then spent $27 on makeup. They were hungry and so spent 1/5 of what they had left on lunch. They then stopped by the music store and bought 4 CDs for $9 each. They spent half of what they had left on a gift for mom. On the way home, they spent $35 on gas. When they got home, they had $87. How much money did they start with?

Step 1: Macy's: $90
Step 2: Shoes: $63
Step 3: Makeup: $27
Step 4: Lunch: 1/5 of what they had
Step 5: CDs: $36
Step 6: Gift: ½ of what they had
Step 7: Gas: $35
End: Money left over: $87

Working backwards, we start with $87 and add $35 to get $122. To undo step 6, we think that they spent half of what they had and ended up with $122. That means they must have had twice that $122 right before step 6. So double $122 to get $244. Undo step 5 by adding $36 to this to get $280. I drew a picture below to illustrate how we figure the cost of lunch to be $70. Add that on and we get to $350. Add the $27, $63, and $90 for the first three steps and you get a grand total of $530 with which they started.

We know they spent 1/5 of what they had on lunch. So draw a rectangle with 5 equal parts. Set aside one to represent the lunch. The other parts must equal $280, the amount they had after lunch. So each piece is $280/4 or $70. That means lunch was also $70.



4a. (3) Use guess-and-check to explore the following question. You do not need to use the table to solve the problem completely because you will be asked to solve it algebraically.
The total cost of a basketball was $19.29, including a 7.25% sales tax. How much of that cost was the price of the basketball and how much was the tax?

Price of ball (before tax)	**Tax (7.25% of price)**	**Total cost (price plus tax)**
$19	.0725*19 = 1.38	$20.38 too high
$18	.0725*18 = 1.31	$19.31 too high
$17.90	.0725*17.90 = 1.30	$19.20 too low

I did not quite get to an answer but I do see that the total cost of the ball is always the price plus the tax (which is 7.25% of the price). That helps form an algebraic equation for the next step.

4b. (3) Use your guess-and-check table from part *a* to now set up and solve an algebraic equation to answer the question. Be sure to explicitly define your variable. Answer the question with a sentence or phrase.

We know, "price of ball" + "tax" = "total cost".

Let x = the price of the ball before tax

So…

$$x + .0725x = 19.29$$
$$1.0725x = 19.29$$
$$\frac{1.0725x}{1.0725} = \frac{19.29}{1.0725}$$
$$x \approx 17.99$$

We use the verbal model "price of ball" + "tax" = "total cost" to form an equation using the variable.

Solve for x and you'll get the price of the ball before tax is about $17.99.

5a. (3) Use guess-and-check to explore the following question. You do not need to use the table to solve the problem completely because you will be asked to solve it algebraically.
Barbie has $4.35 in quarters and nickels. She has three more nickels than she has quarters. How many of each coin does she have?

Number of quarters	**Number of nickels**	**Total amount of money ($)**
10	13	10(.25) + 13(.05) = $3.15 too low
15	18	15(.25) + 18(.05) = $4.40 too high

I know the number of nickels is "3 more than the number of quarters". So I guessed a number of quarters and added 3 to get the appropriate number of nickels. To find the total amount of money (third column), I multiplied .25 times the number of quarters and .05 times the number of nickels, and added those.

5b. (3) Use your guess-and-check table from part *a* to now set up and solve an algebraic equation to answer the question. Be sure to explicitly define your variable. Answer the question with a sentence or phrase.

We know, "amount in quarters" + "amount in nickels" = "total amount of money".

Let q equal the number of quarters. Then, if there are three more nickels than quarters, $q + 3$ must represent the number of nickels.

The equation and its solution are on the next page.

Our equation is to the right. I solved it to find the number of quarters to be 14. So then there must be 17 nickels.

$$.25q + .05(q + 3) = 4.35$$
$$.25q + .05q + .15 = 4.35$$
$$.30q + .15 = 4.35$$
$$.30q = 4.20$$
$$q = 14$$

6. (3) Use the method of finite differences to find the formula for the following function shown in the table below. Write your formula down specifically.

x	$y = \mathbf{f} \cdot x + \mathbf{e}$	**Difference between y-values**
0	13	
1	18	+5
2	23	+5
3	28	+5
4	33	+5
5	38	+5

We see the constant differences column means this is a linear function. So f is 5 and e is 13. The equation they want is

$y = 5x + 13$.

7. (3) Use the formula you found in the last question to find the y-value when x is 37. (If you were unable to find a formula, make up one now and use it to answer the question so you can get these points.) $y = 5(37) + 13 =$ 198

8. (3) Use the method of finite differences to find the formula for the following function shown in the table below. Write your formula down specifically. **Notice that the y-value for $x = 0$ is missing. You will have to infer it from the table.**

x	$y = \mathbf{f} \cdot x + \mathbf{e}$	**Difference between y-values**
0		
1	4	-3
2	1	-3
3	-2	-3
4	-5	-3
5	-8	

We see the constant differences column means this is a linear function. So f is -3. But e is not given on the table. So we need to work our way up the table, inferring that the top y-value is 4+3 or 7. So e is 7.

The equation they want is

$y = -3x + 7$.

9. (3) Use the formula you found in the last question to find the y-value when x is 40. (If you were unable to find a formula, use $y = -5x + 15$ to answer the question so you can get these points.) $y = -3(40) + 7 =$ -113

10. (4) Complete the table of finite differences below, but this time, we do it in general using the function $y = gx^2 + fx + e$. Also find the differences of those differences, shown in the fourth column.

x	$y = gx^2 + fx + e$	**Difference between y-values**	**Difference between entries in previous column (differences)**
0	e		
1	g + f + e	g + f	
2	4g + 2f + e	3g + f	2g
3	9g + 3f + e	5g + f	2g
4	16g + 4f + e	7g + f	2g
5	25g + 5f + e	9g + f	2g

11. (3) Use the method of finite differences to find the formula for the function below. Show your work for finding g, f, and e.

x	$y = gx^2 + fx + e$	**Difference between y-values**	**Difference between entries in previous column (differences)**
0	-3		
1	3	+6	
2	17	+14	+8
3	39	+22	+8
4	69	+30	+8
5	107	+38	+8

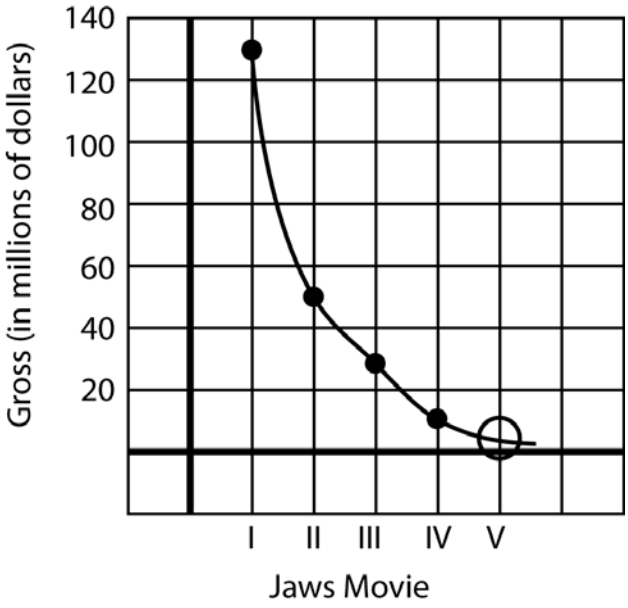
Here, we see that $2g = 8$ so $g = 4$. We see that e is -3. Finally, we see that f + g is 6. Along with the fact that g is 4, this means f must be 2. So the equation is $y = 4x^2 + 2x - 3$.

12. (3) The movie *Jaws* was a big hit and grossed $130 million. The sequel (*Jaws II*) didn't do so well, and neither did *Jaws III* or *Jaws IV*. Using the information in the chart below, draw a graph and use it to predict how much *Jaws V* might have grossed if the producers had filmed it. (There was no fifth film actually made.)

Be sure to label your axes with words and numbers. Write a sentence or two explaining your process.

Movie	*Jaws*	*Jaws II*	*Jaws III*	*Jaws IV*	*Jaws V*
Gross (in millions)	$130	$50	$26	$12	??

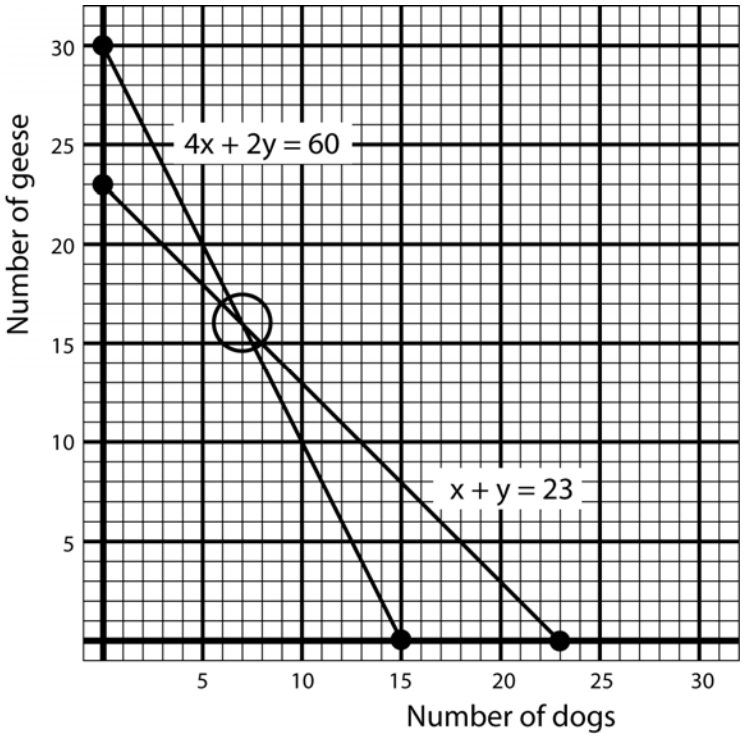
I determined my scale to be 20 for each tick mark for the *y*-axis so that I could fit all of the points. I determined the scale of 1 movie for each tick mark would work nicely for the *x*-axis. I then labeled the axes and plotted the points. I saw this graph had more of a curved pattern, not a straight line. So I drew that curve in, extending it through where the fifth movie would be. Reading off the *y*-value at that point (open circle on right of graph), I would estimate *Jaws V* would gross about $8 million.



13. (3) Stacy has some dogs and geese on her farm. Last week, she bought little shoes for all her animals and so she knows they have a total of 60 feet. Today, she bought them 23 hats (because animals in shoes looked weird and she thought hats would help). How many geese and how many dogs does Crazy Stacy have?

You can solve this problem any way you like. I have provided some graph paper if you choose a graphical method. Label the axes if you use the graph.

I let x represent the number of dogs and y represent the number of geese. I know there are 60 feet, and each dog has 4 feet (presumably) and each goose has 2 feet. So I should multiply the number of dogs by 4 and the number of geese by 2, and add those, to get the total number of feet. So I have the equation $4x + 2y = 60$. Likewise, each animal has one head (presumably; egad!) so we have the second equation
$x + y = 23$.



I graph each by plotting and connecting their x and y-intercepts. Plug 0 in for one variable and solve to find the other axis' intercept (where the graph hits that axis).

The point of intersection of these two lines is the solution to our problem. The point, in ordered point notation, is (7, 16). We read this as (x, y), so remembering that x represents the number of dogs and y represents the number of geese, we know that Crazy Stacy has 7 dogs and 16 geese.

14. (4) We want to make a scale drawing of a room that is 10 feet by 13 feet. If we want to use a scale of 1 centimeter equals 2 feet, answer the following questions.

a.) What are the dimensions of the room on paper (in centimeters)?

b.) In this room, there is a table that is 6 feet by 4.5 feet. What are the dimensions of this table in our scaled drawing?