

Sequence of Squares Exercise (Section 2.4)

NAMES:

Turn in one paper per group but be sure all members of the group have seen the final answers. Circle your name if the paper that gets turned in is your copy.

The following is an exploration based on selected questions from Set II of the exercise set.

A table of the squares of the numbers 1 through 40 is below. Recall the squares are called **square numbers**.

| Number | Square | Number | Square | Number | Square | Number | Square |
|--------|--------|--------|--------|--------|--------|--------|--------|
| 1 | 1 | 11 | 121 | 21 | 441 | 31 | 961 |
| 2 | 4 | 12 | 144 | 22 | 484 | 32 | 1024 |
| 3 | 9 | 13 | 169 | 23 | 529 | 33 | 1089 |
| 4 | 16 | 14 | 196 | 24 | 576 | 34 | 1156 |
| 5 | 25 | 15 | 225 | 25 | 625 | 35 | 1225 |
| 6 | 36 | 16 | 256 | 26 | 676 | 36 | 1296 |
| 7 | 49 | 17 | 289 | 27 | 729 | 37 | 1369 |
| 8 | 64 | 18 | 324 | 28 | 784 | 38 | 1444 |
| 9 | 81 | 19 | 361 | 29 | 841 | 39 | 1521 |
| 10 | 100 | 20 | 400 | 30 | 900 | 40 | 1600 |

Refer to the table to answer the following questions.

1. Can the square of an even number be odd?

2. Can the square of an odd number be even?

The fourth line of the table (which is copied below) suggests that if the last digit (or ones digit) of a number is 4, then the last digit of its square is 6.

| Number | Square | Number | Square | Number | Square | Number | Square |
|--------|--------|--------|--------|--------|--------|--------|--------|
| 4 | 16 | 14 | 196 | 24 | 576 | 34 | 1156 |

3. If the last digit of a number is 7, what is the last digit of its square?

4. For which digits is the last digit of a number the *same* as the last digit of its square?

5. Can a square number end in *any* digit? [In other words, what are the possibilities for the ones digit of a square number? Let's find the possibilities for the ones digit of square numbers together.

a.) To explore this, pick two separate two-digit numbers from 1 to 40 and square them by hand. Show the multiplications by hand.

b.) When we square a number, what determines the digit in which its square ends (in other words, the ones digit of the square)?

c.) To the right we see the squares of the numbers 0 through 9. Considering this list, what are the possibilities for the ones digit of any square number?]

| Number | Square |
|--------|--------|
| 0 | 0 |
| 1 | 1 |
| 2 | 4 |
| 3 | 9 |
| 4 | 16 |
| 5 | 25 |
| 6 | 36 |
| 7 | 49 |
| 8 | 64 |
| 9 | 81 |

6. One of the following numbers is a square and the others are not. Which is the square number? Explain how the realization from question 5 helps us know.

1,372 2,137 3,721 7,213

7. Can a square number of two or more digits have all even digits? If so, give an example.

8. Do you think that a square number of two or more digits can have all odd digits? If so, give an example.

9. [Complete the] table for the squares of the numbers 51 through 58.

| Number | Square |
|---------------|---------------|
| 51 | |
| 52 | |
| 53 | |
| 54 | |
| 55 | |
| 56 | |
| 57 | |
| 58 | |

10. Do you notice a pattern in your table? If so, describe it. [Use the pattern to find the square of 59, without actually doing the calculation.]