Finding Probabilities Involving Sample Means and Proportions NAME:

We know the following about the sampling distribution of the sample mean.

The Shape of the Sampling Distribution of $\bar{x}$ is Normal:
If a random variable X is normally distributed, the sampling distribution of the sample mean $\bar{x}$ is normally distributed.

The Mean and Standard Deviation of the Sampling Distribution of $\bar{x}$:
Suppose that a simple random sample of size n is drawn from a population with mean μ and standard deviation σ. The sampling distribution of $\bar{x}$ has mean $\mu_{\bar{x}} = \mu$ and standard deviation $\sigma_{\bar{x}} = \sigma / \sqrt{n}$. The standard deviation of the sampling distribution of $\bar{x}$, or rather $\sigma_{\bar{x}}$, is called the **standard error of the mean**.

The Central Limit Theorem:
Regardless of the shape of the underlying population, the sampling distribution of $\bar{x}$ becomes approximately normal as the sample size, n, increases. A sample size of 30 is considered enough.

1. The quality control team of a local restaurant wants to reduce the amount of time customers spend in the drive-through window waiting on their food. It has been determined that the mean time spent in the drive-through window is 59.3 seconds with a standard deviation of 13.1 seconds. The distribution of time at the window is skewed to the right.

a.) How many cars should be sampled if we want to use our normal methods to find probabilities related to the mean time spent at the drive-through window for our sample?

b.) Calculate $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \sigma / \sqrt{n}$ for a sample size of 40. Round the mean to one decimal place and the standard deviation to four decimal places.

c.) The restaurant developed a new system to speed up this time. A random sample of 40 cars spent a mean time at the window of 56.8 seconds. What is the probability of obtaining a sample mean of 56.8 seconds or less? Assume the actual mean is 59.3 seconds. Draw and shade an appropriate normal curve. Round to the nearest hundredth of a percent.

d.) If the probability we found in part *c* is unusual, then we can say that the new system has likely made a difference and our assumption that the mean time is really 59.3 seconds is likely flawed. So, is the event “mean time less than or equal to 56.8 seconds” unusual? What do you conclude about the effectiveness of the new system?

e.) Consider the next 40 cars as a random sample. Complete the sentence. Draw a normal curve with the unknown value of X and the area of 5% marked. Round to the nearest whole number.

There is a 5% chance the sample mean will be at or below ___________ seconds.

> If you do this by hand and look up .0500 in the body of Table V, notice there are two *z*-scores equally close, which we average.

We know the following about the sampling distribution of the sample proportion.

Sampling Distribution of the Sample Proportion $\hat{p}$ **:**
For a simple random sample of size n with a population proportion p,

1. The shape of the sampling distribution of $\hat{p}$ is approximately normal *if* $np(1-p) \geq 10$.

2. The mean of the sampling distribution of $\hat{p}$ is $\mu_{\hat{p}} = p$.

3. The standard deviation of the sampling distribution of $\hat{p}$ is $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}}$.

2.) The proportion of booked passengers for a plane who *miss* their flight is known to be 0.0995. Answer the following questions.

a.) If we take a sample of 320 booked passengers, is the shape of the sampling distribution of $\hat{p}$ considered to be normal? Here, we take $\hat{p}$ to be the percentage who *miss* their flight. Explain.

b.) Calculate $\mu_{\hat{p}} = p$ and $\sigma_{\hat{p}} = \sqrt{\dfrac{p(1-p)}{n}}$ for a sample size of 320. Round to four decimal places.

c.) Suppose a flight has 320 booked passengers. However, there are only 300 seats on the plane. (The airline assumes some people will *not* make their flight and so overbook flights.) What is the probability that 300 or less passengers actually make the flight (and so no passengers are bumped from the flight)? Draw and shade an appropriate normal curve. Round to the nearest hundredth of a percent.

Be careful. The value of $\hat{p}$ should be the proportion of people who *miss* their flight. If 300 or less people make the flight, 20 or more miss it.