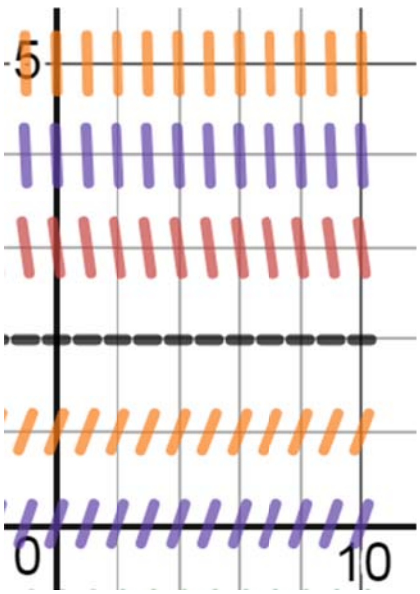
Differential Equations NAME:

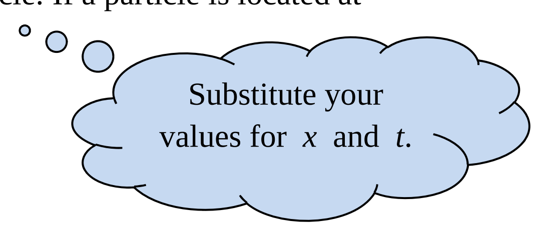
Direction Fields Worksheet

1. Below is the direction field for the differential equation $\frac{dv}{dt} = 1 - \frac{v^3}{8}$. Here, v stands for the velocity, at time t, of an object falling through a viscous medium due to gravity. Sketch the solutions for this differential equation for the initial conditions $v(0) = 0, 1, 2,$ and 3. What is the terminal velocity in all of these cases? (Do not worry about units like feet/seconds.)



2. The motion of a set of particles moving along the x-axis is governed by the differential equation $\frac{dx}{dt} = t^3 - x^3$. Here, $x(t)$ is the position at time t of the particle. Do not worry about units like feet/second.

a.) Recall that this quantity $\frac{dx}{dt}$ would be the velocity of a particle. If a particle is located at $x = 1$ when $t = 2$, what is the velocity of the particle?



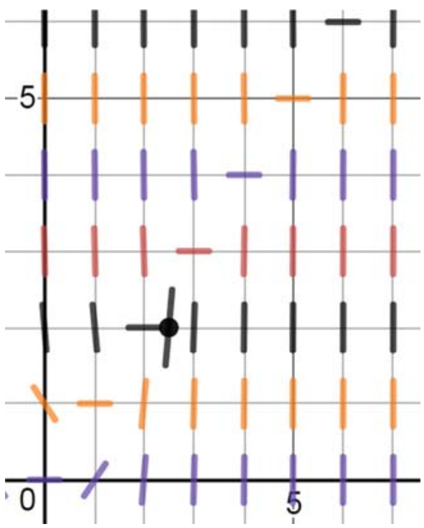
b.) Show that the acceleration of the particle is given by $\frac{d^2x}{dt^2} = 3t^2 - 3t^3x^2 + 3x^5$.

Use $\frac{dx}{dt} = t^3 - x^3$ as given to find the second derivative. You will need to do some additional algebra.

c.) If a particle is located at $x = 2$ when $t = 2.5$, can it reach the location $x = 1$ at a later time? Explain.

For the particle to reach $x = 1$ when it is now at $x = 2$ means that the position function would need to be decreasing. Can you determine if the position function is increasing or decreasing in the interval around the point (2.5, 2)?

d.) Consider the direction field below for this differential equation. I have highlighted the point (2.5, 2) which is drawn along with the slope of the solution curve at that exact point. Notice this slope is positive as you saw in part *c*. This is for your information; there is no question here.



Remember, by solution curve, we mean the function $x(t)$ that spawned the original differential equation.