Differential Equations NAME:

Euler's Method for Approximating Function Values

Complete the problems. Show work but use the online calculator when needed. Do not round the output from the online calculator. Circle final answers.

1. Use Euler's method to find approximations to the solution at $t = 1$ of the initial value problem below. Use the online calculator so you can give these approximations for 1, 2, 4, and 8 steps. [Remember that the online calculator requires the symbol * for multiplication. Also, the variables online are (t, y). You will enter the derivative as $1+t*\sin(t*y)$.] Label each approximation with its value of h. [Recall that h does *not* take on the values 1, 2, 4, and 8 but is related.]

$$\frac{dx}{dt} = 1 + t\sin(tx), \quad x(0) = 0$$

We want to estimate $x(1)$ or the function value when $t = 1$. We do this for various values of h.

2a. Use the strategy shown in class to find a value of h for Euler's method such that $x(1)$ is approximated to within ± 0.01, if $x(t)$ satisfies the following initial value problem. Use the table below to record your $x(1)$ values gotten from the online calculator using the specified values for *h*. You may *not* need to complete the whole table. Write the approximate value of $x(1)$ we are after, rounded to two decimal places, indicating the margin of error of ± 0.01.

$$\frac{dx}{dt} = 1 + x^2, \quad x(0) = 0$$

h	**x(1)**
… (I did not start at $h = 1$ for brevity's sake.)	
1/16	
1/32	
1/64	
1/128	
1/256	
1/512	
1/1024	
1/2048	

2b. Consider the same initial value problem. Use the strategy shown in class to find, to within ± 0.02, the value of t such that $x(t) = 0.8$.

$$\frac{dx}{dt} = 1 + x^2, \quad x(0) = 0$$

2c. It turns out that the solution to this differential equation is $x(t) = \tan(t)$. Verify your answers to parts 2*a* and *b* using this knowledge. Set your calculator to radian mode. Show work.