Differential Equations NAME:

Solving Homogeneous Linear Equations with Real Roots

Solve the following problems, showing your work legibly.

1. Find two distinct solutions in the form $y = e^{rt}$ to the following differential equation. Then show that each solution really does make the original equation true.

$2y'' + y' - 3y = 0$

2. For the differential equation given in number 1, form the general solution as shown in class. Then complete the solution using the initial values $y(0) = 2$ and $y'(0) = 4.5$. That is, solve the initial value problem.

3. We will generalize the method for homogeneous linear second-order differential equations to hold for homogeneous linear third-order differential equations in the form $ay''' + by'' + cy' + dy = 0$. That is, if we find three distinct real roots r_1, r_2, and r_3 for its auxiliary equation, then the functions $y = e^{r_1 t}$, $y = e^{r_2 t}$, and $y = e^{r_3 t}$ are solutions to the differential equation. Find these solutions for the homogeneous linear third-order equation below. Also, give the general solution. (Hint: Solve the auxiliary equation by factoring by grouping.)

$$y''' - 6y'' - y' + 6y = 0$$