We will use exponents a lot as we progress. We need to really understand what they mean and we will learn some important rules.

Elementary algebra
Class notes
Rules of Exponents (section 12.1)

Recall an exponent's purpose. What does 5^3 really mean?

$5^3 = 5 \cdot 5 \cdot 5$

"5 multiplied by itself 3 times"

Can be thought of as "repeated multiplication"

What about $(-2)^4$?

$(-2)^4 = (-2)(-2)(-2)(-2)$

"-2 multiplied by itself 4 times"

What is the value of $(-2)^4$? How is that different than -2^4? The parentheses are very important.

We will be simplifying expressions like $\left(\frac{1}{3}\right)^4$ and $-4 \cdot 3^3$ and $(5y^4)(3y)$. Some can be done by simply thinking about what it means to raise a number by an exponent. Other times it will be easier to use the rules of exponents.

We will start with simplifying (or evaluating) expressions with no variables. Keep the order of operations in mind.

PEMDAS
Parentheses
Exponents
Multiplication/**D**ivision
Addition/**S**ubtraction

expl 1: Evaluate.
-4^3

expl 2: Evaluate.
$(-4)^3$

What difference do the parentheses make?

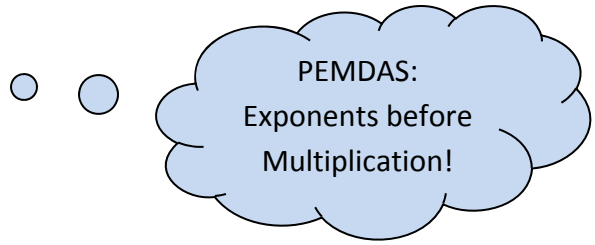
Earlier we saw $(-2)^4$ and -2^4 were different. But $(-4)^3$ and -4^3 were the same. Why?

expl 3: Evaluate.

$$\left(\frac{1}{3}\right)^4$$

expl 4: Evaluate.

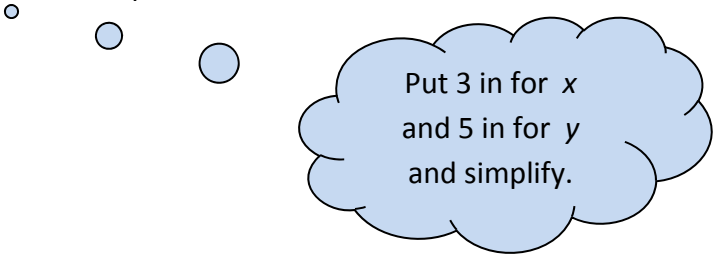
$$-4 \cdot 3^3$$



Now let's throw some variables in there. Keep up the good work with the order of operations.

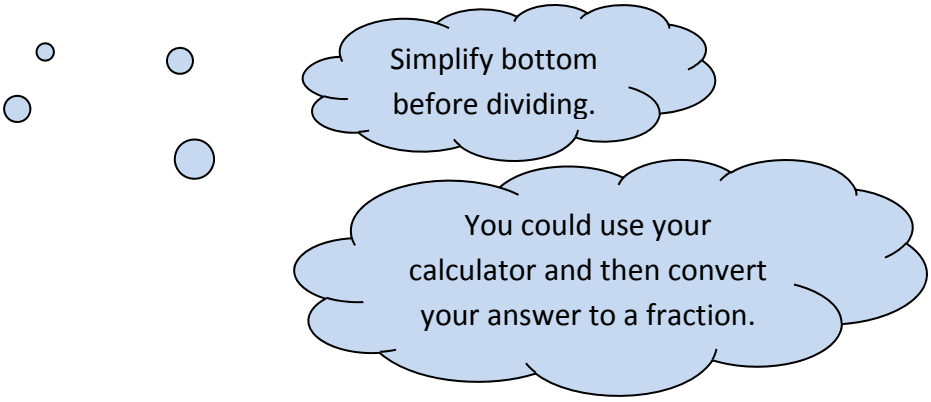
expl 5: Evaluate each expression given the replacement values for the variables.

$2xy^2$; $x = 3$ and $y = 5$



expl 6: Evaluate each expression given the replacement values for the variables.

$\frac{10}{3y^2}$; $y = 5$



Worksheet: Discovering the Rules of Exponents

This worksheet will work on developing the rules from your knowledge of arithmetic and exponents. It will help internalize the rules. They will be easier to use and remember if you do not have to memorize them as meaningless drivel.

You will use the rules of exponents a lot. We may refer to them by name. Complete the rules from memory (after doing the above worksheet) so that we can use them in the next examples. (The variables represent real numbers and denominators are not zero.)

Product rule: $a^m \cdot a^n =$

Quotient rule: $\frac{a^m}{a^n} =$

Power rule: $\left(a^m\right)^n =$

Power of a product rule: $(a \cdot b)^n =$

Power of a quotient rule: $\left(\frac{a}{c}\right)^n =$

Make up quick examples to help. Think about $2^3 \cdot 2^4$.

$2^3 \cdot 2^4 = (2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)$
How many 2's is that? So what is the product rule?

Zero exponent rule: $a^0 =$ (Here a cannot be 0 because 0^0 is undefined.)

expl 7: Use the product rule to simplify each expression. Write the results using exponents.

$x^2 \cdot x^5$

expl 8: Use the product rule to simplify each expression. Write the results using exponents.

$(-3)^3 \cdot (-3)^9$

Do not worry about the final value. Just practice the rule.

expl 9: Use the product rule to simplify each expression. Write the results using exponents.

$(5y^4)(3y)$

Picture the constants together and the variables together.
$5 \cdot 3 \cdot y^4 \cdot y = ?$

expl 10: Use the product rule to simplify each expression. Write the results using exponents.

$(-7a^3b^3)(7a^{19}b)$

expl 11: Use the power rule and the power of a product or quotient rule to simplify.

$(x^9)^4$

Rewrite the rules here before you begin.

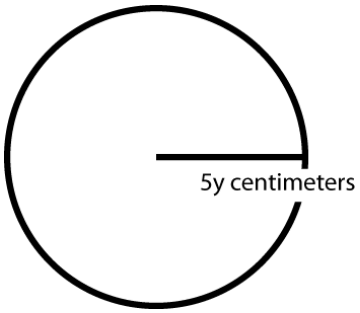
expl 12: Use the power rule and the power of a product or quotient rule to simplify.

$(-7a^2b^5c)^2$

expl 13: Use the power rule and the power of a product or quotient rule to simplify.

$$\left(\frac{xy}{7}\right)^2$$

expl 14: The circle below has a radius of $5y$ centimeters, find its area. Do not approximate π.



Area of a circle with radius r: $A = \pi r^2$

Leave π in place. Do not use 3.14 or the π button on calculator.

Did you attach the correct units to your answer?

You will need to decide which rule to use for many problems. Often you do not need to rely on a rule, but rather could write it out in longhand. I will show that method as we go through these examples.

expl 15: Simplify.

23^0

PEMDAS
E before M!

expl 16: Simplify.

$-2x^0$

expl 17: Simplify.

$\left(\frac{9}{qr}\right)^2$

You could use a rule or write it out as $\left(\frac{9}{qr}\right)\left(\frac{9}{qr}\right)$.

expl 18: Simplify.

a^2a^3a

You could use a rule or write it out as $(aa)(aaa)a$.

expl 19: Simplify.

$(2ab)^5$

How would you write this out in longhand in case you forgot the rule?

expl 20: Simplify.

$\frac{x^{12}y^{13}}{x^5y^7}$

$\frac{x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x \cdot x}{x \cdot x \cdot x \cdot x \cdot x}$

What cancels? What is left?