Expressions with negative exponents will be used more as you progress through algebra and in scientific notation.

Elementary algebra
Class notes
Negative Exponents and Scientific Notation (section 12.5)

Negative exponents:

We have the general rule (or definition) to show us how to interpret a negative exponent.

$a^{-n} = \frac{1}{a^n}$ if a is non-zero and n is an integer

expls: $2^{-3} = \frac{1}{2^3}$ (or $1/8$)

$4^{-2} = \frac{1}{4^2}$ (or $1/16$)

$x^{-3} = \frac{1}{x^3}$

A number raised to a negative exponent is "one over that number to the positive exponent."

Write your own example and check it on your calculator.

This is sometimes hard for students to remember. It might help if we investigate the rule a bit. Let's see how the rules of exponents and fraction simplification work together to make a^{-n} equal to $\frac{1}{a^n}$.

Let's look at $\frac{x^2}{x^5}$. Because of a rule of exponents, we could write that as x^{-3}.

Which rule?

But, also, if we write it out to cancel common factors, we get the following.

$$\frac{x^2}{x^5} = \frac{\cancel{x} \cdot \cancel{x}}{\cancel{x} \cdot \cancel{x} \cdot x \cdot x \cdot x} = \frac{1}{x \cdot x \cdot x} = \frac{1}{x^3}$$

So x^{-3} must be equal to $\frac{1}{x^3}$.

We also have $\frac{1}{a^{-n}} = a^n$ (if a is non-zero and n is an integer). So if you have a number raised to a negative exponent on the bottom of a fraction, you can rewrite it on top and make the exponent positive. We'll see that in a few examples.

expl 1: Simplify the expression. Write your answer with positive exponents only.

$7x^{-3}$

> Remember that the -3 only applies to the *x*, not the 7 too, because of how exponents work.

We will be using the rules of exponents a lot in this section. Try to fill them in from memory. (The variables represent real numbers and denominators are not zero.)

Product rule: $a^m \cdot a^n =$

> $2^3 \cdot 2^4 = (2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)$
>
> How many 2's is that? So what is the product rule?

Quotient rule: $\dfrac{a^m}{a^n} =$

Power rule: $(a^m)^n =$

Power of a product rule: $(a \cdot b)^n =$

Power of a quotient rule: $\left(\dfrac{a}{c}\right)^n =$

Zero exponent rule: $a^0 =$ (Here a cannot be 0 because 0^0 is undefined.)

And add our new rules, $a^{-n} =$ and $\dfrac{1}{a^{-n}} =$ (if $a \neq 0$ and n is an integer).

expl 2: Simplify the expression. Write your answer with positive exponents only.

$$\frac{2}{x^{-4}}$$

expl 3: Simplify the expression. Write your answer with positive exponents only.

$$\frac{a^{-3}}{a}$$

expl 4: Simplify the expression. Write your answer with positive exponents only.

$$\frac{-2}{t^{-3}}$$

> Remember that the -3 only affects what happens to the *t.*

expl 5: Simplify the expression. Write your answer with positive exponents only.

$$\frac{y^4 y}{y^{-2}}$$

expl 6: Simplify the expression. Write your answer with positive exponents only.

$$\frac{\left(t^{-2}\right)^5}{t^{-3}t^2}$$

expl 7: Simplify the expression. Write your answer with positive exponents only.

$$\frac{\left(3x^2y^{-2}\right)^{-2}}{(xy)^{-3}}$$

> I would start off with $\frac{(xy)^3}{\left(3x^2y^{-2}\right)^2}$.
>
> Why does that work?

Scientific Notation:

Scientific notation is used to write really small numbers like .000 000 000 000 645 or really big numbers like 7,000,000,000,000,000 in shorthand notation.

Spaces help count zeros, like commas in big numbers.

Main idea: Since 1,000,000,000,000,000 is 10^{15} (see argument at top of next page), and 7,000,000,000,000,000 is 7 times that, we could write this enormous number as 7 x 10^{15}.

One quadrillion

Convention says we use an "x" multiplication sign, but you do *not* have to, especially if x is used as a variable.

Likewise, .000 000 000 000 645 could be written simply as 6.45 x 10^{-13}.

Notice small numbers get negative exponents.

Optional worksheet: Scientific notation and your calculator
This worksheet works on the powers of 10 and also shows how scientific notation is displayed and inputted on the TI calculators.

Definition: Scientific notation: A positive number is written in scientific notation if it is written as the product of a number *a*, where $1 \le a < 10$, and an integer power *r* of 10: a x 10^r

This means *a* must be greater than or equal to 1 but less than 10.

What are the integers?

{... -3, -2, -1, 0, 1, 2, 3, ...}

Some examples:
a.) 5,000 is written as 5 x 10^3 since 5,000 is equal to 5 x 1,000 (and 1,000 is the same as 10^3).

b.) .0008 is written as 8 x 10^{-4} since .0008 is equal to 8 x .0001 (and .0001 is the same as 10^{-4}).

Why not 80 x 10^{-5} or .8 x 10^{-3}?

As described in the optional worksheet, scientific notation is based on the decimal system. See below for a discussion of the patterns used in scientific notation.

$$\vdots$$

$$10^{-3} = .001$$

$$10^{-2} = .01$$

$$10^{-1} = .1$$

$$\boxed{10^{0} = 1}$$ Start here

$$10^{1} = 10$$

$$10^{2} = 100$$

$$10^{3} = 1000$$

$$\vdots$$

Subtracting 1 from exponent adds another 0 to right side.

Negative exponent equals number of decimal places in number on right side.

Adding 1 to exponent adds another 0 to right side.

Positive exponent equals number of zeros in number on right side.

If you continue this pattern, you can see why 1,000,000,000,000,000 is 10^{15}.

expl 8: Write in scientific notation.

1,160,000

Procedure:
1. Move decimal point until you get a number between 1 and 10,
2. Count the number of spaces you moved (positive if you moved left, negative if you moved right),
3. Write answer as number from step 1, times 10 to the power of the count in step 2.

expl 9: Write in scientific notation.

.000 000 17

Book writes it as 0.00000017. We can ignore the zero to the left of the decimal point (called a leading zero). Also, use spaces to keep track of zeros like commas in big numbers.

expl 10: Write in scientific notation.

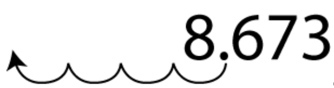
.00194

Did you get a negative power of 10?

expl 11: Write in standard notation (decimal form).

8.673×10^{-10}

Negative exponent means it's a really small number.



Move decimal point to left 10 places. Fill in with zeros.

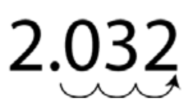
expl 12: Write in standard notation (decimal form).

3.3×10^{-2}

expl 13: Write in standard notation (decimal form).

2.032×10^{5}

Positive exponent means it's a really large number.



Move decimal point to right 5 places. Fill in with zeros.

Don't forget your commas.

expl 14: Light travels at a rate of 1.86×10^5 miles per second. The distance from the sun to Venus is roughly 67,000,000 miles. How long will it take light from the sun to reach Venus? Round your answer to the nearest second. (Hint: Use the formula $d = r \cdot t$ where d is distance, r is rate, and t is time.)

Work the formula and record what we need here. Follow calculator instructions below.

Calculator note: Look for **EE** on your calculator. It's the 2nd function of the comma on most TI calculators. Let's do the previous problem using the calculator. Enter the following.

67000000 [÷] 1.86 [EE] 5 [ENTER]

2nd function of comma

Notice how scientific notation is displayed on your calculator. Sometimes, an answer will be outputted with that little **E** and you want to know what it means.

Worksheet: Multiplying Polynomials and Negative Exponents:

This worksheet will help you practice using the distribution property to multiply polynomials as well as the ideas from this section.