Function notation and the concept of function will follow you throughout algebra.

Elementary Algebra
Class Notes
Functions (section 3.6)

Idea behind Functions:

Equations like $y = 4x + 5$ or $x^2 + 3y = 16$ show relationships between variables. They tell us how x and y are related. These are called **relations**. They can also be represented by a table of values, a list of ordered pairs, or a graph which is just a picture of those ordered pairs. A function is a special kind of relation. Let's cover some terminology.

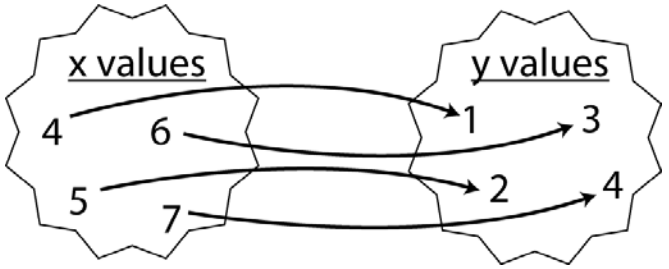
Definition: Domain: the set of all x values (that will give you a real number out for y)

Definition: Range: the set of all y values (that you can get out for y)

x-values: inputs
y-values: outputs

expl 1: Consider the sets of ordered pairs and their illustrations below. Determine the domain and range of these relations.

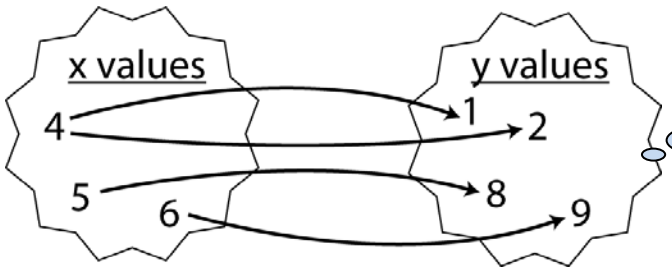
a.) (4, 1), (5, 2), (6, 3), and (7, 4)



Each x value is assigned to a **specific y value.**

What is the domain? What is the range? Write your answers in set notation.

b.) (4, 1), (4, 2), (5, 8), and (6, 9)



The x value of 4 is assigned to **two different y values**.

What is the domain? What is the range? Write your answers in set notation.

Definition: Function: a relation where every *x* value in the domain is assigned to **exactly one *y* value**.

In example 1 above, is the relation in part *a* a function? Is the relation in part *b* a function? Explain.

expl 2: Which of the following relations are functions?

Make a table of values.

a.)

x	***y***
1	1
2	2
3	3
4	4

b.) (4, 5)
(5, 6)
(6, 7)
(7, 4)
(4, 7)

c.) $y = x^2$

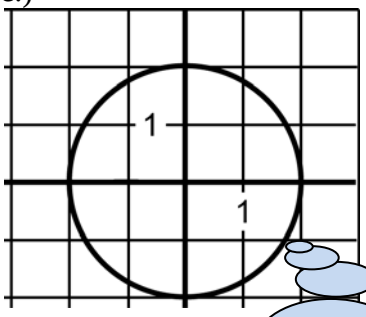
x	***y***
-2	
0	
2	
4	

d.) $x = y^2$

x	***y***
0	
1	
4	
9	

Complete the table of values.

e.)



Highlight the points for the *x* value of 1. Does that help?

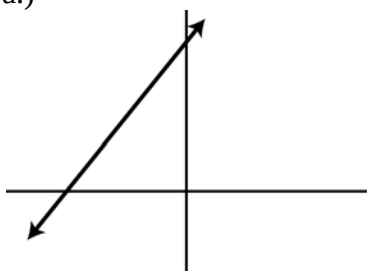
f.) $y = 3x + 5$

Vertical Line Test: Given a graph, the vertical line test will tell you if it is a function. If any vertical line could be drawn so that it crosses the graph more than once, then it is **not** a function. (The vertical line represents a single *x* value. If this vertical line hits the graph more than once, that *x* value has more than one *y* value and so the relation is **not** a function.)

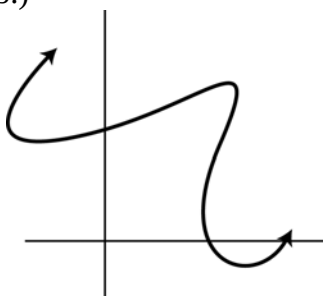
Try the vertical line test on part *e* above.

expl 3: Use the vertical line test to determine if the following are functions.

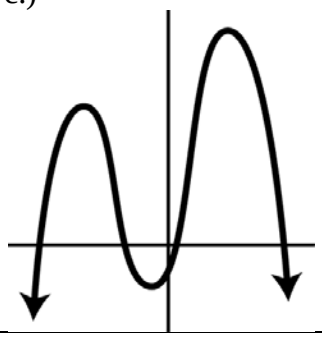
a.)



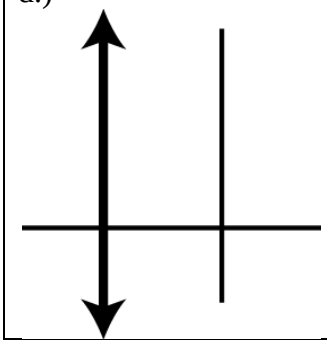
b.)



c.)



d.)



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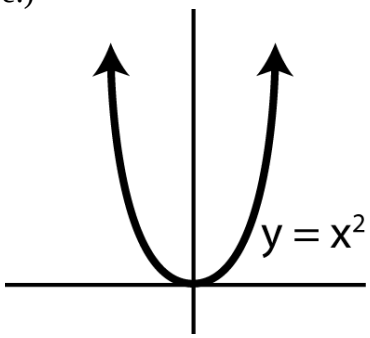
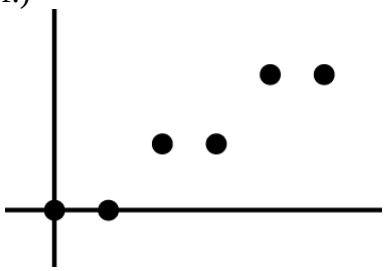


f.)



Interpretation: You can think of a function in a few different ways.

1. a **relationship** between two variables, x and y,
2. a **rule** that tells you what to do to an x value to get out a y value, or
3. a **machine** that produces a y value when you input an x value.

In certain applications, one understanding of function may serve us better than the others.

$y = 2x^2 + 4$

Relationship: Every y value is equal to the x value squared, multiplied by 2, and added to 4.

Rule: This $y = 2x^2 + 4$ tells us what to do to an x value to make a y value. Can you picture a **machine** doing these operations?

Function notation:

Check to see if the following relationships are functions.

Does any x value result in more than one y value?

$y = 4x + 5$		$y = 2x^2 + 4$		$y^2 = x$ (or $y = \pm\sqrt{x}$)	
x	$y = 4x + 5$	x	$y = 2x^2 + 4$	x	$y^2 = x$ (or $y = \pm\sqrt{x}$)
-3		-3		9	
0		0		16	
3		3		25	
Is y a function of x?		Is y a function of x?		Is y a function of x?	

Since the first and second relationships are functions, we can use function notation to make sure everyone knows. So we replace the y with $f(x)$ to write $f(x) = 4x + 5$ or $g(x) = 2x^2 + 4$. Sometimes we use different letters like $g(x)$ or $h(x)$, especially if we have more than one function.

We say y is a "function of x".

pronounced "f of x"

expl 4a: Find $g(0)$, $g(-2)$, and $g(5)$ for the function $g(x) = 2x^2 + 4$.

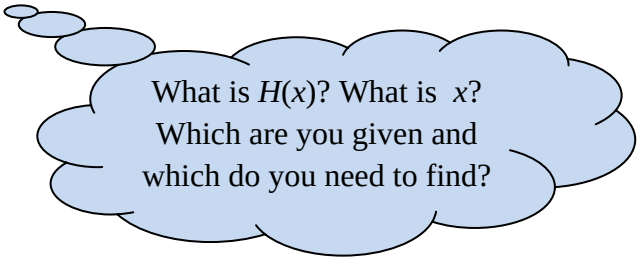
How do you square -2 on the calculator?

What we want are the y values when x is 0, -2, and 5.

expl 4b: Recall that the numbers 0, -2, and 5 are x values and the $g(x)$ outputs are their corresponding y values. Write your results from part a in ordered pair notation.

Common Mistakes with Notation: As we use function notation in more complicated ways, understanding the notation and using it correctly will be of utmost importance. For instance, in the previous example, we must **never write** $g(x) = 54$ or $g(5) = 2x^2 + 4$. Whatever you write in the parentheses should be substituted for x in the formula at the same time.

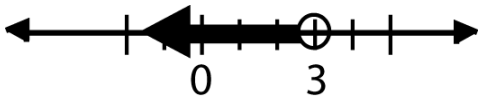
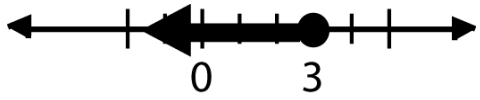
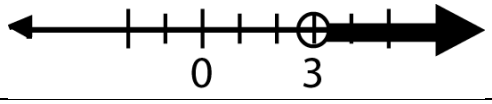
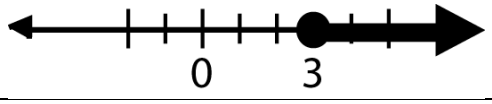
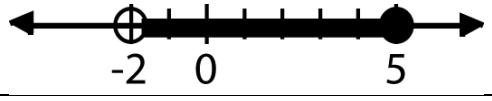
expl 5: Forensic science uses the function $H(x) = 2.59x + 47.24$ to estimate the height $H(x)$ of a woman (in centimeters) given the length x (in centimeters) of her femur bone. Estimate the height of a woman whose femur bone measured 46 cm.



Optional Worksheet: "Investigating functions" gives you practice determining if a relationship is a function and using function notation.

Review of Interval Notation:

Do you remember interval notation? Fill in the third column for these sets of numbers. The real number line graphs can help visualize the sets.

Inequality Notation	Graph on Number Line	Interval Notation
$x < 3$		
$x \leq 3$		
$x > 3$		
$x \geq 3$		
$-2 < x \leq 5$		

"the numbers in between -2 and 5, not including -2 but including 5"

smallest number in set , largest number in set

square bracket: includes endpoint
parenthesis: does not include endpoint

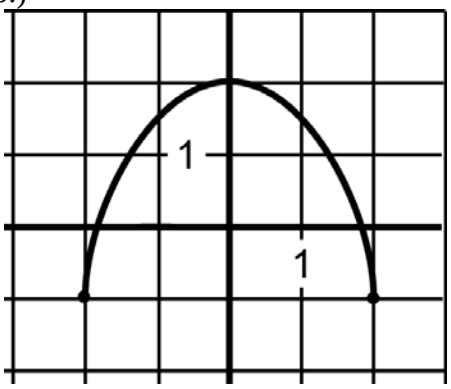
Finding domain and range:

Which is x values?
Which is y?

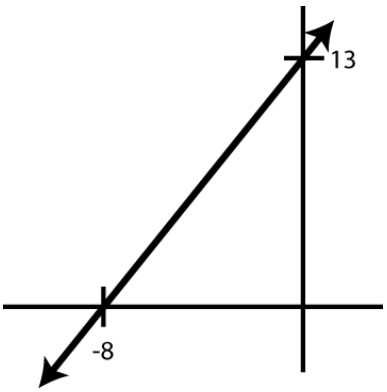
expl 6: Find the domain and range for the various functions. Use interval notation or set notation where appropriate.

a.) (1, 0), (2, 3), (4, 6), and (5, 6)

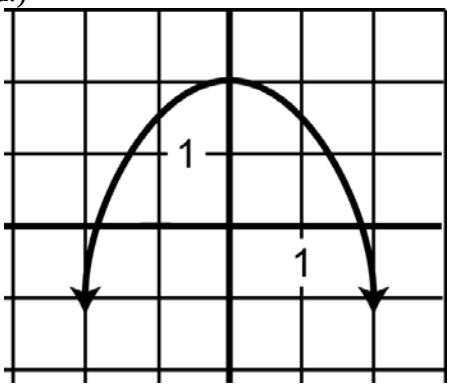
b.)



c.)



d.)



e.) $y = \dfrac{3}{x+4}$

f.) $h(x) = |x|$

Domain: Instead of asking what x could be, ask yourself what x cannot be. The domain is everything else.

Range: What values can you get out for y? What values could you *never* get out for y?