

Elementary algebra
 Class notes
 Linear Inequalities and Problem Solving (section 9.6)

We will work these problems the same as we would equations, with one minor change.

Recall Linear Equation: an equation that could be written in the form $ax + b = c$ where a is not zero.

We now have **Linear Inequality:** an inequality that could be written in the form $ax + b < c$ where a is not zero.

less than $<$
 greater than $>$
 How do you remember which is which?
 What about \leq and \geq ?

We will solve them similarly with one exception. Do you know what the exception is? We will investigate this by experimenting with the true inequality $4 < 10$. Perform each operation below to see if the inequality is still true. The first is started for you.

$4 < 10$	$4 < 10$	$4 < 10$	$4 < 10$	$4 < 10$
Divide by -2	Multiply by 2	Add 5	Subtract 7	Multiply by -1
↓	↓	↓	↓	↓
$\frac{4}{-2} ? \frac{10}{-2}$				

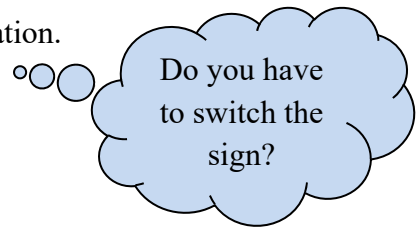
Again, here is a true inequality $-4 > -10$. Think about it to convince yourself it is true and then complete the operations below. Are the resultant inequalities also true?

$-4 > -10$	$-4 > -10$	$-4 > -10$	$-4 > -10$	$-4 > -10$
Divide by -2	Multiply by 2	Add 5	Subtract 7	Multiply by -1
↓	↓	↓	↓	↓
$\frac{-4}{-2} ? \frac{-10}{-2}$				

So which operations made the inequalities untrue? What do we need to do to correct for this?

expl 1: Solve. Graph the solution set and write it in interval notation.

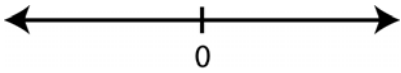
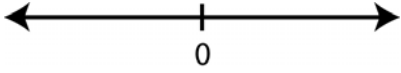
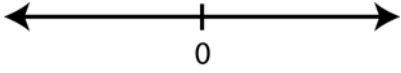
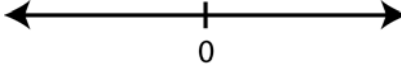
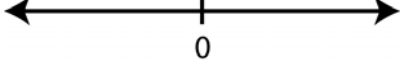
$$2x < -6$$

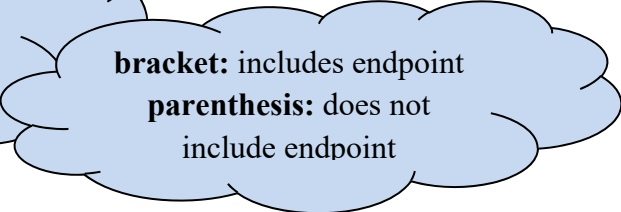
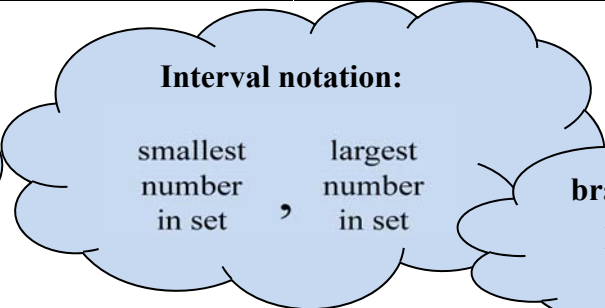
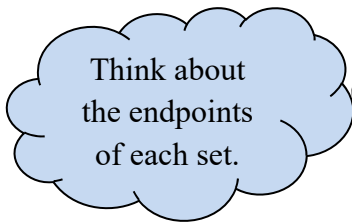


Check your answer by plugging a number less than -3 into the original inequality? Does it make it true? Now plug in a value greater than -3 ; is the inequality true?

Interval Notation and Graphs on the Number Line:

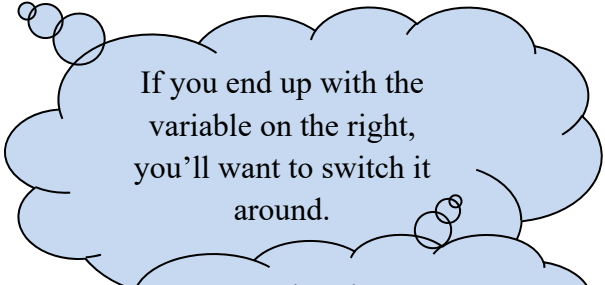
Let's talk about how we might represent a set of numbers like those less than -3 . Graphing on the real number graph can help visualize the set.

Inequality Notation	What it means	Graph on Number Line	Interval Notation
$4 < x < 10$	the numbers in between 4 and 10, including neither		
$-3 \leq x < 5$	the numbers in between -3 and 5, including -3 but not 5		
$x > 5$	the numbers greater than 5		
$x < -3$	the numbers less than -3		
$x \leq 10$	the numbers less than or equal to 10		

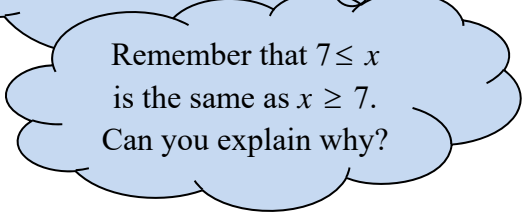


expl 2: Solve. Graph the solution set and write it in interval notation.

$$3x + 9 \leq 5(x - 1)$$



If you end up with the variable on the right, you'll want to switch it around.



Remember that $7 \leq x$ is the same as $x \geq 7$. Can you explain why?

expl 3: Solve. Graph the solution set and write it in interval notation.

$$6(2 - x) \geq 12$$

Take the time to check your answer by putting a few values (some in your solution set and some not) into the inequality.

Double or Compound Inequalities:

expl 4: Solve. Graph the solution set and write it in interval notation.

$$1 < 4 + 2x \leq 7$$

What operations would you do to solve $4 + 2x = 7$? **Do the same here but to all three parts of the inequality.**

If you prefer, you can solve $1 < 4 + 2x$ and $4 + 2x \leq 7$ separately, and then combine the solutions.

Applications:

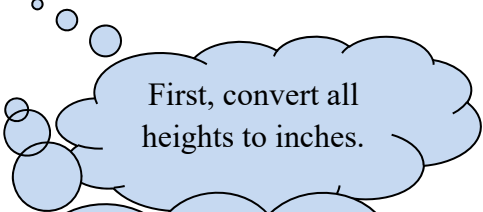
expl 5: Bob and JoAnn have \$2500 to spend on their wedding reception. If the banquet hall charges a flat cleanup fee of \$75 plus \$35 per guest, what is the maximum number of people they can invite?

Let x represent the number of guests. Find the cost for x guests. Set up the inequality "cost ≤ 2500 ".

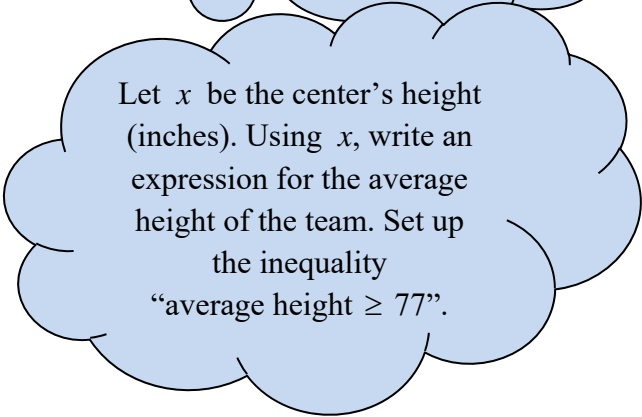
Make sure your answer makes sense in context.

What will their total bill be?

expl 6: A certain WNBA team has two forwards measuring 6'8" and 6'6" and two guards measuring 6'0" and 5'9". How tall a center should they hire if they wish to have a starting team average height of at least 6'5"?



First, convert all heights to inches.



Let x be the center's height (inches). Using x , write an expression for the average height of the team. Set up the inequality "average height ≥ 77 ".

Now check your answer by finding the average height of the team using the center's minimum possible height.

Worksheet: Inequalities and you 1:

This worksheet investigates the phenomenon of switching the sign when we divide or multiply by a negative. It also provides practice solving inequalities including double inequalities.