Elementary algebra
Class notes
Multiplying Polynomials (sections 12.3 and 12.4)

We will use my favorite property, the distribution property. We will also learn about FOIL.

Do you remember the distribution property? Try to write it down using variables. Then write an example of it using actual numbers.

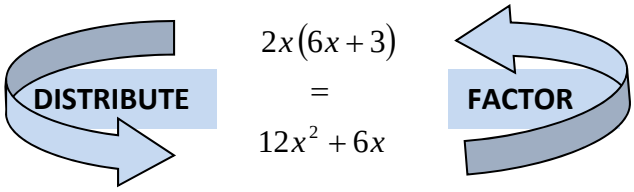
The distribution property is my favorite because it is used so much. It gives us the ability to turn a product into a sum or turn a sum into a product. I wrote it down along with an example below. Did you remember it correctly?

$$a \cdot (b + c) = a \cdot b + a \cdot c$$
$$3 \cdot (2 + 5) = 3 \cdot 2 + 3 \cdot 5$$

product
"3 **times** 7"

sum
"6 **plus** 15"

Notice how the two sides of the equation are in fact equal. Does your example work out too? We will use this property to multiply things like $2x(6x + 3)$. We would "distribute the $2x$" to get our answer. In future sections, we will go the opposite way. That will be called "factoring". The relationship between the distribution property and factoring is illustrated below.



Try some examples to practice the distribution property.

Remember those exponent rules!

expl 1: Multiply.
$9t^6(-3t^5)$

Why does the distribution property **not** apply here?

expl 2: Multiply.

$-2a^2(3a^2 - 2a + 3)$

Simplify as you go.

expl 3: Multiply.

$-y(4x^3 - 7x^2y + xy^2 + 3y^3)$

Convention says write the x first in mixed terms.

expl 4: Multiply.

$$\frac{1}{2}x^2\left(8x^2 - 6x + 1\right)$$

FOIL (**F**irst **O**utside **I**nside **L**ast)

mnemonic device used to multiply 2 binomials

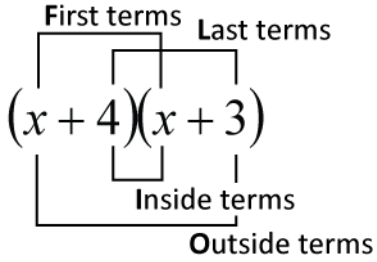
To understand that this is not some trick, **we will multiply (x + 4)(x + 3) without relying on FOIL**. We will see how FOIL is simply a mnemonic device for this multiplication.

Multiply (x + 4)(x + 3).

Start by distributing (x + 4) through the whole x + 3.

Your first line should be $(x + 4)\cdot x + (x + 4)\cdot 3$. How would you continue?

So $(x+4)(x+3) = \underline{\underline{x \cdot x}} + \underline{\underline{4 \cdot x}} + \underline{\underline{3 \cdot x}} + \underline{\underline{3 \cdot 4}}$. This happens to be the sum of the products of the **first** terms in each binomial, the **inside** terms, the **outside** terms, and the **last** terms in each binomial. See the picture below.



Simplify $(x+4)(x+3) = \underline{\underline{x \cdot x}} + \underline{\underline{4 \cdot x}} + \underline{\underline{3 \cdot x}} + \underline{\underline{3 \cdot 4}}$ to write the product in finished form.

In the future, use FOIL to multiply two binomials. Do it enough and it becomes second nature. Remember it does not matter which order you write your terms. Always simplify by combining any like terms to get the finished answer.

Optional Worksheet: Understanding FOIL

This worksheet uses pictures to help you understand why FOIL works to multiply two binomials. It is a convincing argument that FOIL is needed for expressions like $(2x + 3)^2$.

expl 5: Use FOIL to multiply.

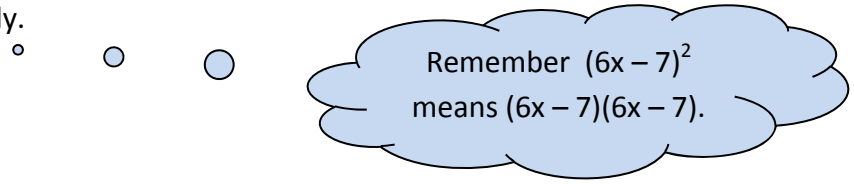
(a + 7)(a – 2)

expl 6: Use FOIL to multiply.

$(3x^2 + 1)(4x^2 + 7)$

expl 7: Use FOIL to multiply.

$(6x - 7)^2$



expl 8: Use FOIL to multiply.

$(x^2 + 4)^2$

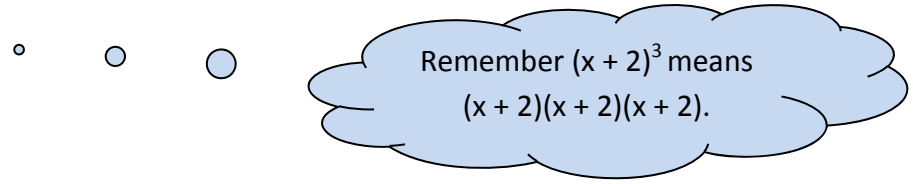
You may be asked to multiply a binomial and a trinomial or two trinomials. How would you work the following problems?

expl 9: Multiply.

$(x - 2)(x^2 - 3x + 7)$

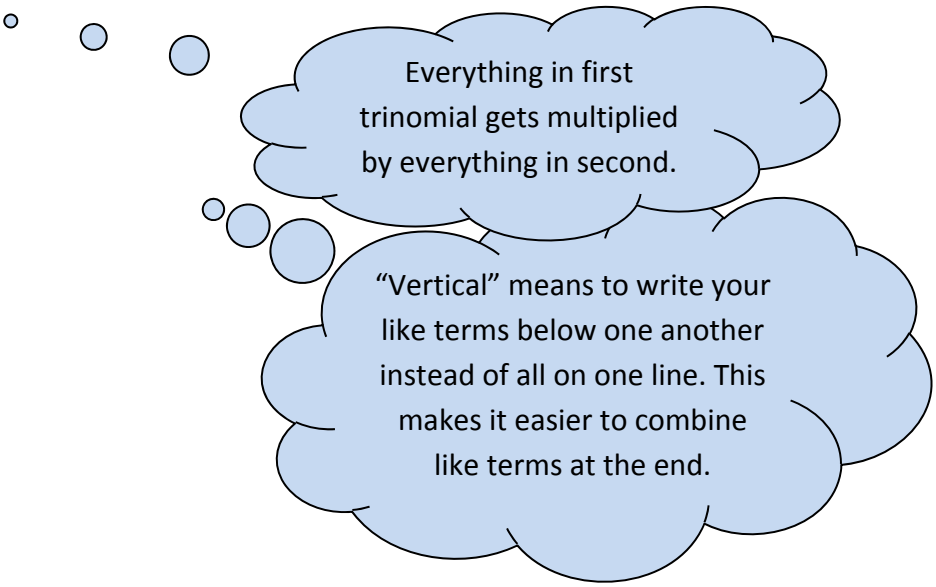
expl 10: Multiply.

$(x + 2)^3$



expl 11: Multiply vertically.

$(x^2 + 5x - 7)(2x^2 - 7x - 9)$



expl 12: Multiply using FOIL.

$(x - 5)(x + 10)$

expl 13: Multiply using FOIL.

$(2x - 9)(x - 11)$