We will turn two equations with two unknowns into one equation with one unknown.

Elementary Algebra
Class Notes
Algebraically Solving Systems of Linear Equations by Substitution (section 4.2)

Consider our dilemma. We are given two equations but we can't solve either for x because that darn y is in the way. If only we had just one equation with just one variable. Then we could solve it like we are used to. Look at the example here.

expl 1: Solve the system.

$$3x + 4y = 18$$
$$y = -\frac{1}{2}x + 4$$

But, isn't that y in the first equation the same as the y in the second equation? How could we make use of that?

Do you know why this method is called substitution yet? Have you found x yet?

Remember the solution is an ordered pair in the form (x, y). Once you find x, how would you find y that goes along with it? Do it now. Write your solution as an ordered pair.

The Substitution Method:

Solve one of the equations for one of the variables (which may already be done for you),
substitute that into the other equation to form one equation with one variable,
solve for that lone variable,
substitute that value into one of the original equations to find the other variable,
write your solution as an ordered pair,
check your solution by making sure it does make both original equations true, and
dance, baby, dance!

expl 2: Solve by the substitution method.

$$y = 2x + 3$$
$$5y - 7x = 18$$

expl 3: Solve by the substitution method.

$$3y - x = 6$$
$$4x + 12y = 0$$

Which equation and which variable is easiest to solve for?

expl 4: Solve by the substitution method.

$$2y = x + 2$$
$$6x - 12y = 0$$

It is easiest to think of the second equation as $\mathbf{6x - 6 \cdot 2y = 0}$. Why?

What was the solution last time we got a result like this?

expl 5: Solve by the substitution method.

$$\frac{1}{4}x - 2y = 1$$
$$x - 8y = 4$$

What was the solution last time we got a result like this?

Some texts will write the solution to number 5 as $\left(x, \frac{1}{8}x - \frac{1}{2}\right)$. Here, I solved one of the equations for y and wrote that in the (x, y) form. We, on the other hand, will simply say the system has "infinitely many solutions" and be done with it.

Foreshadowing the Addition Method:

Try the following problems to prepare for the next method of solving these systems.

expl 6: Write an equivalent equation by multiplying by 5.

$-2x + y = 12$

expl 7: Add the following polynomials.

$$\begin{array}{r} -3x+8y \\ +\quad 3x+15y \\ \hline \end{array}$$

expl 8: Write an equivalent equation by multiplying by –2.

$-2x + y = 12$