Intermediate algebra
Class notes
Radicals and Radical Functions (section 17.1)

These are square roots, cube roots, etc.

Square roots: We say that the square root of 16 is 4. We write this as $\sqrt{16} = 4$.

Root symbol is a "radical".

Technically, the square root of a number is the number that, when squared, makes the original number. Since $4^2 = 16$, we say 4 is the square root of 16. Can you think of another number whose square is 16?

We call 4 the "**principal square root**" of 16. It's the **non-negative square root**. And -4 is called the "negative square root". In the future, when we talk about the "square root" of a number, we are talking about just the principal or non-negative root.

Why not just say "positive"?

It is **not** correct to write $\sqrt{16} = \pm 4$. The radical symbol means the principal (non-negative) root.

expl 1: Simplify (or evaluate) the following.

a.) $\sqrt{64}$

b.) $\sqrt{\frac{4}{81}}$

Think of how you multiply fractions.

c.) $\sqrt{-25}$

What squared makes -25?

You may remember finding $\sqrt{-25}$ to be $5i$. This is a complex number and is explained in the notes in a later section. For now, we will simply say $\sqrt{-25}$ is "*not* a real number" and leave it at that.

Other roots:

Cube root: The cube root of a real number a is written $\sqrt[3]{a}$ and is defined to be the number you cube (multiply by itself three times) to get the original a. We could say $\sqrt[3]{a} = b$ if $b^3 = a$. Can you think of an example?

Fourth root: The fourth root of a real number a is written $\sqrt[4]{a}$ and is defined to be the number you raise to the fourth power to get the original a.

We continue like that…

index: the number in the crook of the radical. If there is no number, it is assumed to be 2.

In general, the ***n*th root** is written $\sqrt[n]{a}$. Here, n stands for any natural number greater than one (2, 3, 4, …). It could mean square root, cube root, tenth root, twentieth root, or any other root. We use $\sqrt[n]{a}$ to talk about rules that apply to all roots, regardless of the index.

Calculator usage:

The square root symbol is the 2nd function of the $\boxed{x^2}$ button, located on the left side of the calculator. Other roots are found under the **MATH** menu. You will find $\sqrt[3]{\ }($ and $\sqrt[x]{\ }$ under the **MATH** menu. Look for them now and then exit out of the menu. Do the examples below using the calculator.

expl 2: Simplify (or evaluate) the following.

a.) $\sqrt{500}$

Press: 2nd button, $\boxed{x^2}$ button, 500, $\boxed{\text{ENTER}}$

b.) $\sqrt[3]{216}$

Press: $\boxed{\text{MATH}}$ button, arrow down to $\sqrt[3]{\ }($, 216, $\boxed{\text{ENTER}}$

c.) $\sqrt[7]{2{,}097{,}152}$

Press: 7, $\boxed{\text{MATH}}$ button, arrow down to $\sqrt[x]{\ }$, 2097152, $\boxed{\text{ENTER}}$

Round decimals to 3 decimal places

So, we found that the seventh root of 2,097,152 is 8, or $\sqrt[7]{2{,}097{,}152} = 8$. What does that mean? Write this information using an exponent. Check it on the calculator.

Finding $\sqrt[n]{a^n}$:

How would you simplify $\sqrt{x^2}$? Let's investigate this by using your calculator. We will then generalize about $\sqrt[n]{a^n}$ **.** Notice the use of double parentheses for negative numbers.

a.) $\sqrt[3]{5^3}$ $\qquad$ $\sqrt[3]{\ }(5 \wedge 3)$ $\qquad$ e.) $\sqrt{5^2}$

b.) $\sqrt[3]{(-5)^3}$ $\qquad$ $\sqrt[3]{\ }((-5) \wedge 3)$ $\qquad$ f.) $\sqrt{(-5)^2}$

c.) $\sqrt[4]{8^4}$ $\qquad$ $4\ \sqrt[x]{\ }(8 \wedge 4)$ $\qquad$ g.) $\sqrt[7]{8^7}$

d.) $\sqrt[4]{(-8)^4}$ $\qquad$ $4\ \sqrt[x]{\ }((-8) \wedge 4)$ $\qquad$ h.) $\sqrt[7]{(-8)^7}$

So, if n is an **odd** integer (like 3 or 7), what would you say $\sqrt[n]{a^n}$ is equal to?

But, if n is an **even** integer (like 2 or 4), what would you say $\sqrt[n]{a^n}$ is equal to?

To be clear, how would you simplify $\sqrt{x^2}$? Can you say it is just x ?

You will use this rule when you simplify stuff like $\sqrt[3]{x^3}$ and $\sqrt{100x^2y^4}$. Notice the distinction shown above only matters if the radicand is negative. If a is positive or zero, the rule is simply $\sqrt[n]{a^n} = a$, no matter if n is odd or even.

radicand: the number under the radical expl 3: Find each root. Assume the variables are real numbers.

a.) $\sqrt[3]{x^3}$

Since the variable could be a negative number, include absolute value signs in your answer if needed.

b.) $\sqrt{100x^2}$

expl 4: Find each root. Assume the variables are positive real numbers.

a.) $\sqrt[3]{27b^3}$

Since the variable could **not** be a negative number, you do **not** need to include the absolute value signs.

b.) $\sqrt{100x^2y^4}$

expl 5: Find each root. Assume the variables are non-negative real numbers.

a.) $\sqrt[3]{-27b^3}$

What cubed makes -27?
What cubed makes b^3?

b.) $\sqrt[4]{-16}$

c.) $\sqrt[5]{\dfrac{x^5}{243y^{10}}}$

d.) $\sqrt{x^2+6x+9}$ (Hint: Factor the inside!)

Radical functions:

expl 6: Let $f(x) = \sqrt{3x+4}$. Find $f(20)$.

Watch the order of operations! The rule says to "multiply by 3, add 4, and then take the square root". So do that to your x value.

Domain of radical functions: The domain of a function is the set of x values that work, or that when inputted, give you a value out for y.

For even root functions, the radicand (the number under the radical symbol) must be non-negative so you have to solve "radicand $\geq$ 0" to find its domain. **The domain of any even root function is all real numbers where the radicand is non-negative.**

But for odd root functions, this restriction does *not* exist. **So the domain of any odd root function is "all real numbers".**

Think about the distinction here. We could find $\sqrt[3]{-27b^3}$ but we could *not* find $\sqrt{-25}$.

Worksheet: Domain of radical functions:
This worksheet will help you make sense of the domain of radical functions. We will use the calculator to graph these functions to explore their domain (x values) and range (y values).

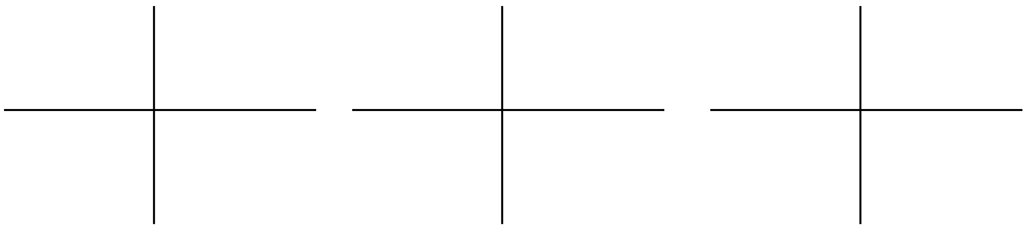
Graph radical functions on the calculator: We want to get used to how the graphs of radical functions look. We will also investigate their domains.

expl 7: Graph the following functions on your calculator. Use the standard window. Notice their general shapes are similar but their x-intercepts vary. Copy the graphs here, recreating the shape and x-intercepts as accurately as you can.

$y=\sqrt{x}$

$y=\sqrt{x-2}$

$y=\sqrt{2x+6}$



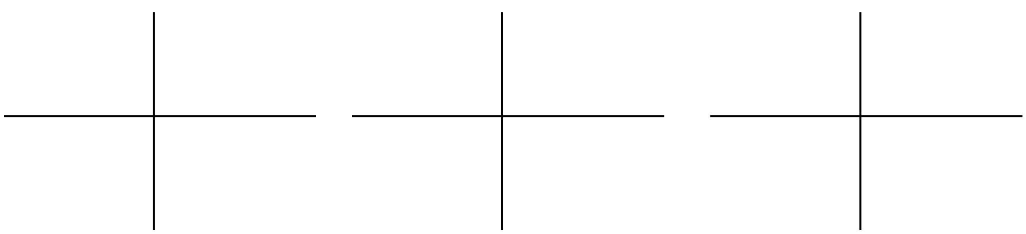
Function:	$y=\sqrt{x}$	$y=\sqrt{x-2}$	$y=\sqrt{2x+6}$
Domain:			

expl 8: Graph the following functions on your calculator. Use the standard window. Notice their general shapes are similar but their x-intercepts vary. Copy the graphs here, recreating the shape and x-intercepts as accurately as you can.

$y=\sqrt[3]{x}$

$y=\sqrt[3]{x-2}$

$y=\sqrt[3]{2x+6}$



Function:	$y=\sqrt[3]{x}$	$y=\sqrt[3]{x-2}$	$y=\sqrt[3]{2x+6}$
Domain:			

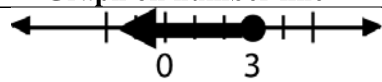
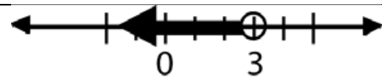
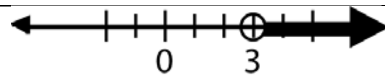
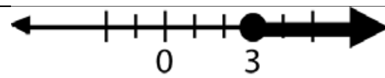
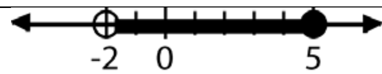
The shapes of even root functions and odd root functions are quite different. The preceding worksheet investigates this.

Can you write the domains of the functions you drew above?

Let's consider range (or y-values). Why is it that y *cannot* be -1 for $y=\sqrt{x}$ but can be for $y=\sqrt[3]{x}$?

Interval notation: Interval notation is a way to write continuous set of numbers. Below I have a table that shows the inequality notation and the interval notation for several sets. I have also drawn the graphs on real number lines. The graphs help me picture the sets, making writing the interval notation easier.

"smallest number, comma, largest number"

Inequality notation	Graph on number line	Interval notation
$x \le 3$		$(-\infty, 3]$
$x < 3$		$(-\infty, 3)$
$x > 3$		$(3, \infty)$
$x \ge 3$		$[3, \infty)$
$-2 < x \le 5$		(-2, 5]

This means "the numbers between -2 and 5, not including -2 but including 5".

Parenthesis used if endpoint is **not** included.

Bracket used if endpoint **is** included.

Parentheses are **always** used next to ∞ or $-\infty$.

expl 9: Fill in the missing parts of the table below.

Inequality notation	Graph on number line	Interval notation
$5 < x < 12$		
		$(-\infty$, -5)
$x \le 54$		
		$[10, \infty)$