Technology Integrated Mathematics
Class Notes
Algebra: Scientific Notation (Section 7.8)

The Milky Way is 1,000,000,000,000,000,000 km across. There's got to be easier way to say that!

Scientific notation is used to write really small numbers like 0.000 000 000 000 645 or really big numbers like 7,000,000,000,000,000 in shorthand notation.

Main idea: Since 1,000,000,000,000,000 is 10^{15} (see argument below), and 7,000,000,000,000,000 is 7 times that, we could write this enormous number as 7×10^{15}.

Spaces help count zeros, like commas in big numbers.

One quadrillion

Convention says we use an "×" multiplication sign, but you do *not* have to, especially if x is used as a variable.

Likewise, 0.000 000 000 000 645 could be written simply as 6.45×10^{-13}.

Notice small numbers get *negative* exponents.

Scientific notation is based on the decimal system. See below for a discussion of the patterns used in scientific notation.

$\vdots$
$10^{-3} = .001$
$10^{-2} = .01$
$10^{-1} = .1$
$10^{0} = 1$ Start here.
$10^{1} = 10$
$10^{2} = 100$
$10^{3} = 1000$
$\vdots$

Subtracting 1 from exponent adds another 0 to right side.

Negative exponent equals number of decimal digits in number on right side.

Adding 1 to exponent adds another 0 to right side.

Positive exponent equals number of zeros in number on right side.

If you continue this pattern, you can see why 1,000,000,000,000,000 is 10^{15}.

Exponent Rules:
In our discussion, we may use a few rules we have not seen yet.

This tells us that 10^{-3} is just $1/10^3$ which is $1/1000$.

Zero Exponent Rule: $a^0 = 1$ for any non-zero base a

Negative Exponent Rule: $a^{-n} = \frac{1}{a^n}$ for any non-zero base a

***Product Rule:** $a^m \times a^n = a^{m+n}$

Think about how $2^3 \cdot 2^4 = (2 \cdot 2 \cdot 2)(2 \cdot 2 \cdot 2 \cdot 2)$ How many 2's is that? See how the product rule gets the same answer?

***Quotient rule:** $\frac{a^m}{a^n} = a^{m-n}$

* These rules are only used when calculating by hand.

We may need these rules, but first, let's get to the definition of a number in scientific notation.

Definition: Scientific Notation:
A number is expressed in scientific notation if it is written in the form $P \times 10^k$ where P *is some number greater than or equal to 1 but also less than 10*. The number k will be an integer (meaning from the set {… -2, -1, 0, 1, 2, …}.)

Some examples:
a.) 5,000 is written as 5×10^3 since 5,000 is equal to $5 \times 1{,}000$ (and 1,000 is the same as 10^3).

b.) 0.0008 is written as 8×10^{-4} since 0.0008 is equal to 8×0.0001 (and 0.0001 is the same as 10^{-4}).

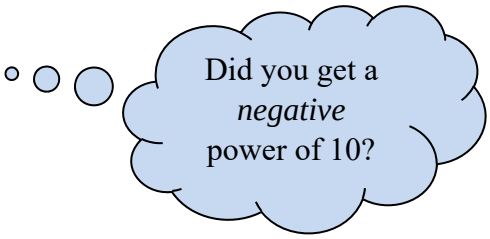
Why *not* 80×10^{-5} or 0.8×10^{-3}?

expl 1: Write in scientific notation.
1,160,000

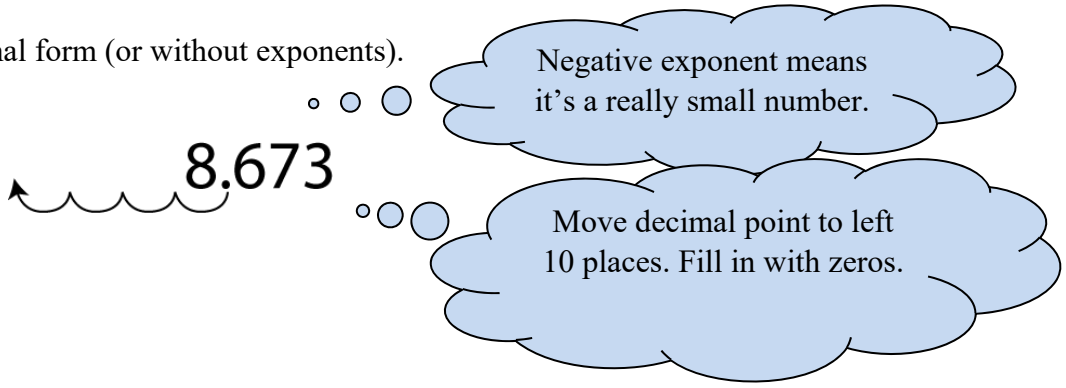
Procedure:
1. Move decimal point until you get a number between 1 and 10,
2. Count the number of spaces you moved (positive if you moved left, negative if you moved right),
3. Write answer as number from step 1, times 10 to the power of the count in step 2.

expl 2: Write in scientific notation.
0.000 000 17

expl 3: Write in scientific notation.
0.00194

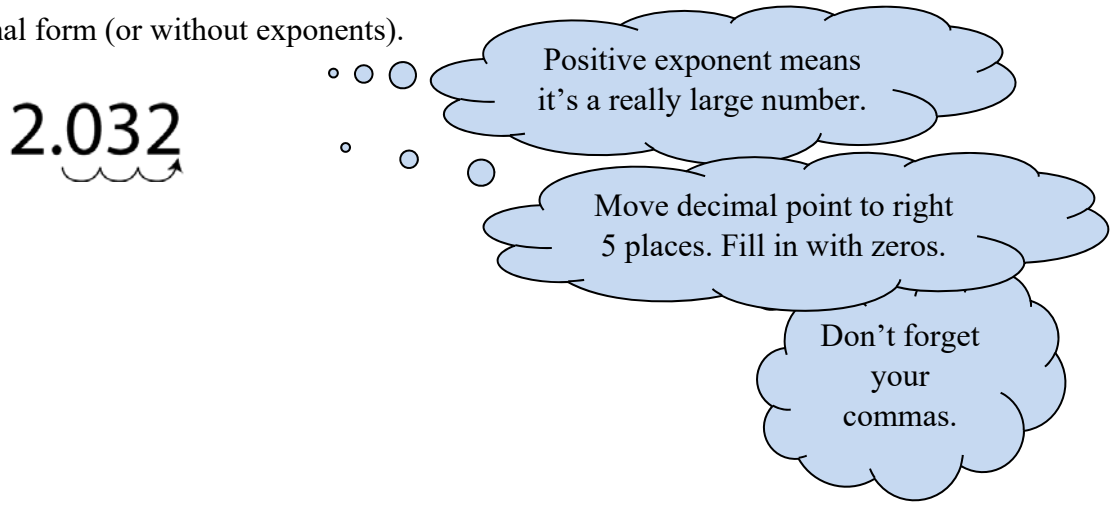


expl 4: Write in decimal form (or without exponents).
8.673×10^{-10}



expl 5: Write in decimal form (or without exponents).
3.3×10^{-2}

expl 6: Write in decimal form (or without exponents).
2.032×10^{5}



Calculators:
Calculators use various ways to display scientific notation. Enter the product of two really large numbers and it will likely default to scientific notation. Do so now and you will see how your calculator displays these numbers.

The TI (Texas instruments) graphing calculators use **E** near the end of the line. This symbol separates the P and the k values from the definition given earlier.

Entering a number in scientific notation will require some button but it varies by calculator. The book suggests it may look like **EE** or **EXP** or $\times$**10^{x}**.

expl 7: Perform the operation on the calculator. The book would have you do it by hand using the product rule shown for exponents earlier. Write the final answer in scientific notation, rounded to one decimal place.
a.) $460{,}000 \times 0.0017$

b.) $\dfrac{0.0000056}{0.00023}$

expl 8: A brick wall 15 m by 25 m is 0.48 m thick. Under particular temperature conditions, the rate of heat flow through the wall, in calories per second, is given by the expression $H = \left(1.7 \times 10^{-4}\right) \times \left(\dfrac{1500 \times 2500}{48}\right)$. Calculate this value to the nearest tenth. Include units.