Technology Integrated Mathematics
Class Notes
Decimals: Converting Between Decimals and Fractions (Section 3.3)

Rational numbers are fractions whose decimal forms either repeat or terminate.

Here, we will convert between the decimal and fraction forms of numbers. The book talks about dividing two numbers and analyzing the remainder, assuming the division is done by hand. We will see what this looks like on the calculator.
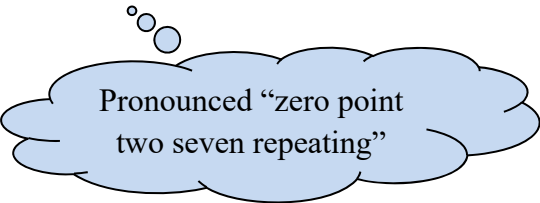
There are two things that can happen when you divide two numbers (assuming the divisor is *not* 0, which gives us a third thing). The decimal form can either terminate or repeat.

Definition: Terminating decimal: a decimal number whose digits just stop like 0.8 or 0.45
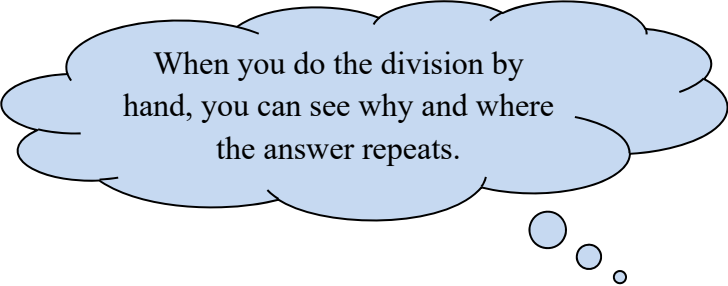
A fraction equals a terminating decimal if you get *no* remainder when you do the division by hand. On the calculator, we will see an answer that does *not* go on and on or repeat itself.

Definition: Repeating decimal: a decimal number whose digits go on forever, repeating all or just a part, like $\frac{1}{3} = 0.3333333...$ or $\frac{3}{11} = 0.27272727...$ or $\frac{11}{30} = 0.3666666...$ This is only limited by the number of digits the calculator can display.

That ellipsis (the … symbol) at the end is not very precise so we use another way to denote when the digits repeat. We will write a "bar" over the repeated digits making the numbers above $0.\overline{3}, 0.\overline{27},$ and $0.3\overline{6}$. Notice, you have to be precise where you place the bar.

Pronounced "zero point two seven repeating"

When you do the division by hand, you can see why and where the answer repeats.
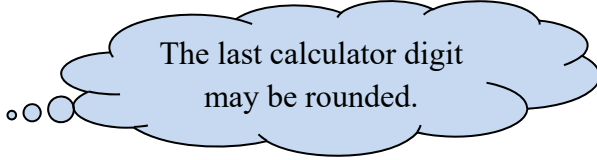
expl 1: Write the following as decimal numbers. Do *not* round. Use the bar notation for repeating decimals.

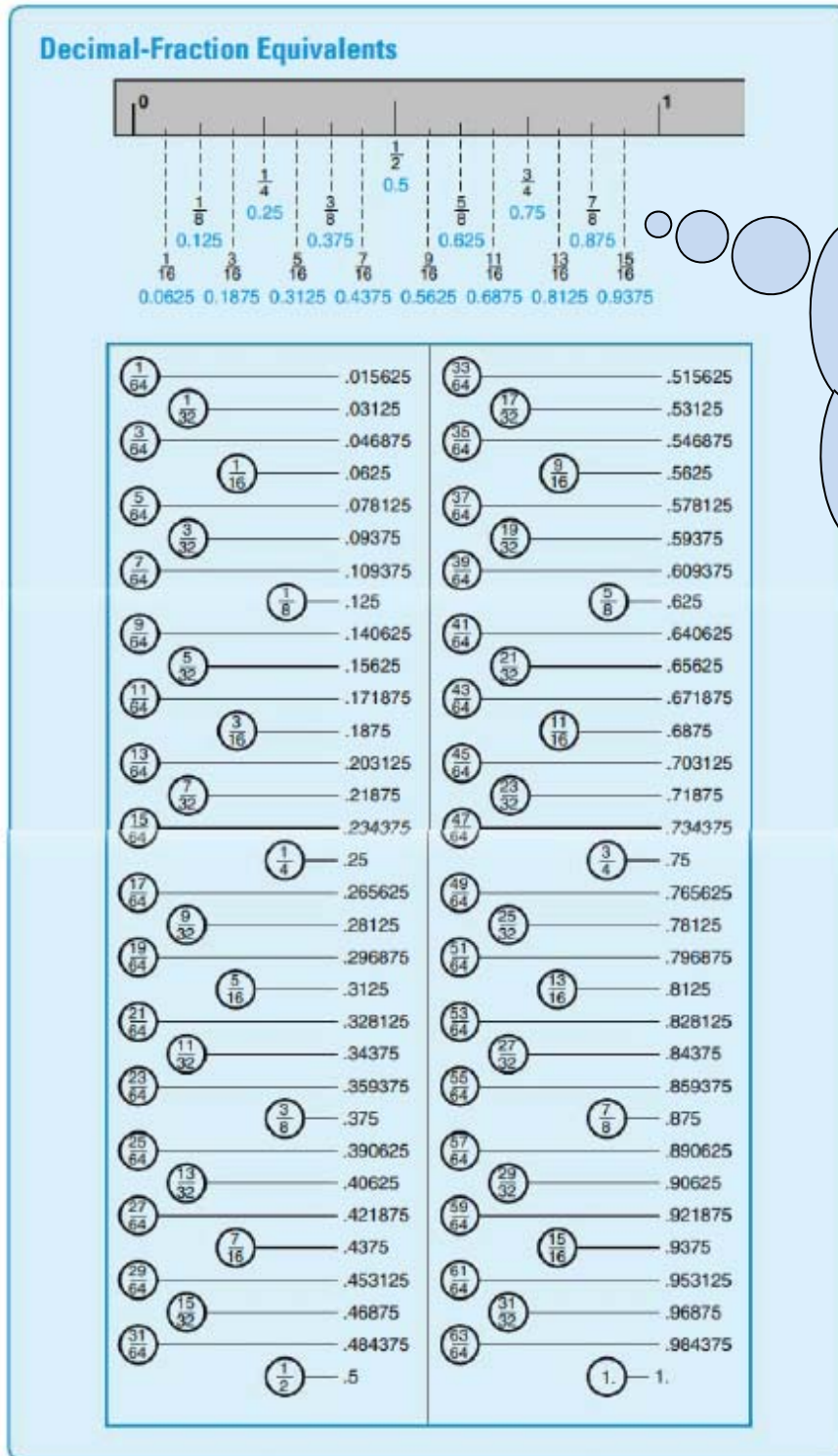a.) $\frac{1}{4}$ b.) $\frac{5}{11}$
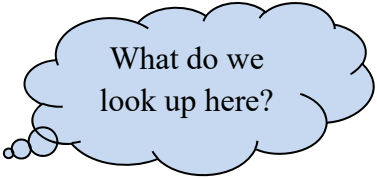
The last calculator digit may be rounded.

c.) $\frac{6}{7}$ d.) $\frac{3}{7}$

1

Shop Fractions:

These are common fractions used in the trades. Below is a graphic showing common denominators of 2, 4, 8, 16, 32, and 64. This is helpful as you can look up commonly used fractions to find their equivalent decimal forms.

Decimal-Fraction Equivalents

The number of decimal digits is consistent. All "halves" have one decimal digit. All quarters have two digits. All eighths have three digits. All sixteenths have four digits. Etcetera.

Fraction	Decimal		Fraction	Decimal
1/64	.015625		33/64	.515625
1/32	.03125		17/32	.53125
3/64	.046875		35/64	.546875
1/16	.0625		9/16	.5625
5/64	.078125		37/64	.578125
3/32	.09375		19/32	.59375
7/64	.109375		39/64	.609375
1/8	.125		5/8	.625
9/64	.140625		41/64	.640625
5/32	.15625		21/32	.65625
11/64	.171875		43/64	.671875
3/16	.1875		11/16	.6875
13/64	.203125		45/64	.703125
7/32	.21875		23/32	.71875
15/64	.234375		47/64	.734375
1/4	.25		3/4	.75
17/64	.265625		49/64	.765625
9/32	.28125		25/32	.78125
19/64	.296875		51/64	.796875
5/16	.3125		13/16	.8125
21/64	.328125		53/64	.828125
11/32	.34375		27/32	.84375
23/64	.359375		55/64	.859375
3/8	.375		7/8	.875
25/64	.390625		57/64	.890625
13/32	.40625		29/32	.90625
27/64	.421875		59/64	.921875
7/16	.4375		15/16	.9375
29/64	.453125		61/64	.953125
15/32	.46875		31/32	.96875
31/64	.484375		63/64	.984375
1/2	.5		1.	1.

expl 2: Use the table to convert the following to decimal form.

a.) $\frac{1}{8}$ b.) $\frac{24}{64}$ c.) $2\frac{3}{4}$

What do we look up here?

We can also use the diagram to find the fraction that is closest to a given decimal measurement.

expl 3: Use the table to find the closest fractional equivalent to 0.425.

Find the two fractions that bracket this decimal.

Which one is closer to 0.425?
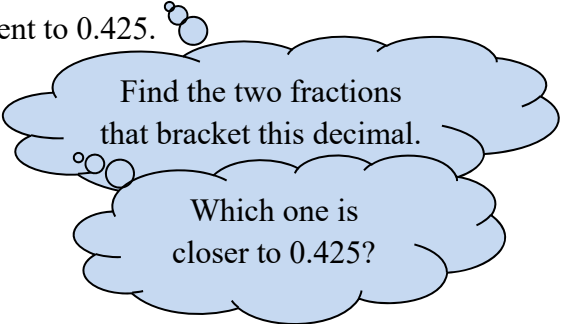
What if we were using a ruler that only had marks every 1/8 of an inch? We could *not* make use of the answer 27/64, could we? We would need to answer the following question instead.

expl 4: Use the table to find the closest fractional equivalent to 0.425 to the nearest 1/8 of an inch.

Do the same as above but only consider fractions with bottoms of 2, 4, or 8.
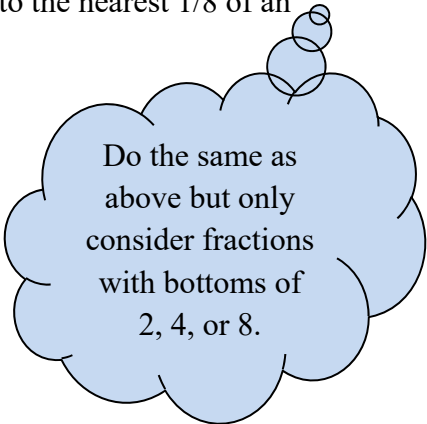
An Important Caveat:
We found the fractional equivalent that is *closest* to 0.425. However, in practical applications, we often choose the *larger* size rather than the closest. Imagine needing to do this to find a wrench. You would *not* want to try the wrench that was just a little too small, would you?

Denominators of 10, 100, 1000 etc.:

Remember that you can simply move the decimal point to the left for division by the powers of 10.

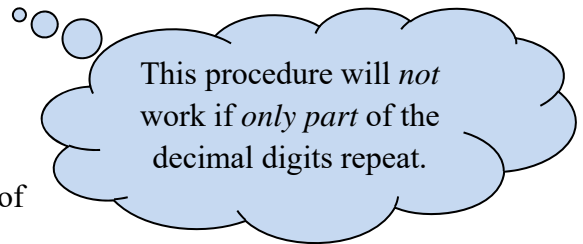expl 5: Use the tip above to write the following as decimals.

a.) $\frac{7}{10}$

b.) $\frac{7}{100}$

c.) $\frac{7}{1,000,000}$

This is also helpful for fractions like $\frac{4}{5}$ as we can see it is equal to $\frac{8}{10}$. Convert that to decimal form.

Converting Repeating Decimals to Fraction Form:

If the entire decimal portion repeats, like $0.\overline{3}$ or $0.\overline{27}$ but *not* like $0.3\overline{6}$, then we can use this quickie method.

> This procedure will *not* work if *only part* of the decimal digits repeat.
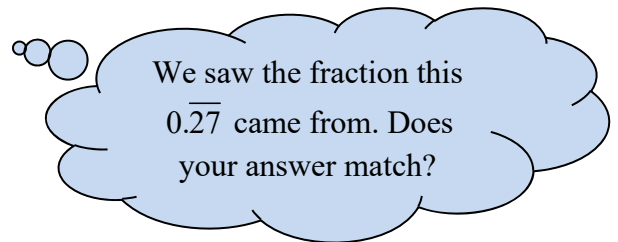
Step 1: Use the digits that repeat as the top (numerator) of the fraction.

Step 2: Use the same number of consecutive 9's for the bottom (denominator).

Step 3: Reduce to lowest terms if needed.

expl 6: Convert the following to fraction form. You may need to reduce your answer. Check on the calculator.

a.) $0.\overline{27}$

> We saw the fraction this $0.\overline{27}$ came from. Does your answer match?

b.) $0.\overline{4}$

c.) $0.\overline{123}$

4

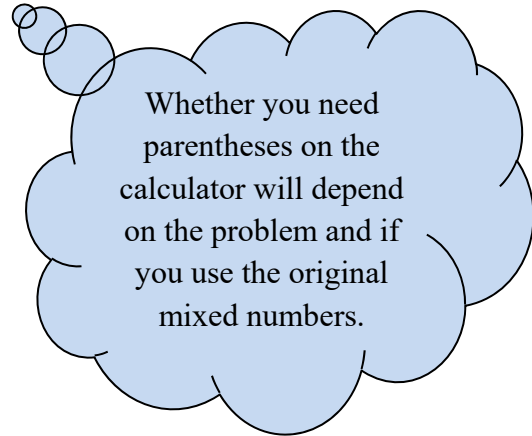**Calculating with Both Fractions and Decimal Numbers:**

Consider the problem $2\frac{3}{5}+1.785$. It is probably easiest to convert to decimal form and add. We can do this on the calculator. You will round to the number of decimal places given in the decimal number.

expl 7: Calculate in decimal form. Round to agree with the number of decimal digits given in the decimal number.

a.) $2\frac{3}{5}+1.785$

Whether you need parentheses on the calculator will depend on the problem and if you use the original mixed numbers.
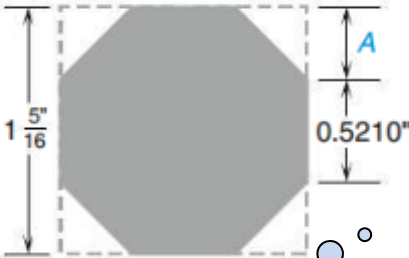
b.) $3\frac{4}{5}\div 2.65$

expl 8: One tablet of calcium pantothenate contains 0.5 gram. Answer the following questions.

a.) How much calcium pantothenate is contained in $2\frac{3}{4}$ tablets?

b.) How many tablets are needed to make up 2.6 grams?

5

expl 9: A square will have its corners cut out to make an octagon as shown. Find the corner measurement A needed. Round to four decimal places.

$1\frac{5"}{16}$

A

0.5210"

What do we do *after* we subtract? Think about the physical situation.