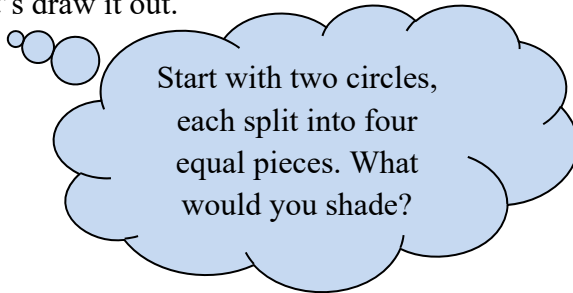


Proper versus Improper Fractions:

Definitions: A **proper fraction** is one where the top is less than the bottom. An **improper fraction** is one where the top is greater than or equal to the bottom.

We call them improper because it is a little hard to think about. We saw (and hopefully understood) $\frac{4}{5}$ earlier. But what would $\frac{5}{4}$ look like? How can we select five out of four parts?

Let's draw it out.



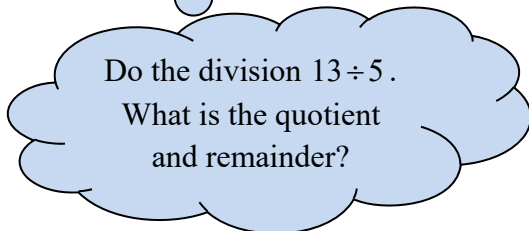
Notice that we have a full circle shaded plus one part of the second circle. We could write this as $1 + \frac{1}{4}$ or, more conveniently, $1 \frac{1}{4}$. This is a **mixed number**.

Converting from Improper Fractions to Mixed Numbers:

Besides drawing them out, how do we turn improper fractions into mixed numbers? We will do the division, determining the remainder as well. Let's see an example.

expl 2: Write as a mixed number.

$$\frac{13}{5}$$



We will use the following general formula.

$$\text{mixed number} = \text{quotient} + \text{remainder} / \text{original denominator}$$

expl 3: Write as a mixed number.

$$100/6$$

Do the division $100 \div 6$.
What is the quotient and remainder?

We will need to know how to reduce (simplify) fractions like $\frac{4}{6}$.

Converting from Mixed Numbers to Improper Fractions:

expl 4: Write $15 \frac{2}{3}$ as an improper fraction.

We will see this denominator in our answer.

We want to know how many “thirds” this is equal to. So, find how many “thirds” are in the 15 and then add 2.

Calculation-wise: We take the 15 times 3 and then add 2 to get the numerator for our improper fraction. Again, the denominator will be the denominator of the fractional part of the mixed number.

expl 5: Write $6 \frac{4}{7}$ as an improper fraction.

Equivalent Fractions and Reducing Fractions:

Definition: Equivalent fractions are fractions that represent the same number.

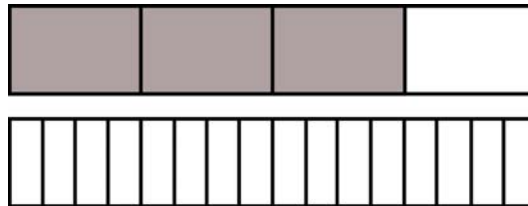
Look at the following fraction bars. One represents $\frac{2}{3}$ and the other represents $\frac{4}{6}$. Label which is which. These are **equivalent fractions**.



expl 6: Complete.

$$\frac{3}{4} = \frac{?}{16}$$

The top fraction bar represents $\frac{3}{4}$. Shade the bottom so it is equal and use that to answer the question.



So, how we deal with these arithmetically, without drawing out pictures every time?

We are going to use the fact that if we multiply the bottom of a fraction by some number, we change the value of the fraction. Right? However, if we *also multiply the top by that same number*, we do *not* change its value.

So, look at those fractions again. We have $\frac{3}{4} = \frac{?}{16}$. We notice that if we multiply the 4 on bottom by 4, we would get 16, right? So, let's multiply the top by that same 4 to get 12. We have created an equivalent fraction, $\frac{12}{16}$!

expl 7: Complete.

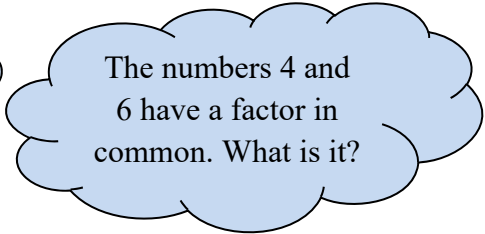
$$2\frac{5}{6} = \frac{?}{12}$$

First, convert the mixed number to an improper fraction.

Reducing to Lowest Terms or Simplifying Fractions:

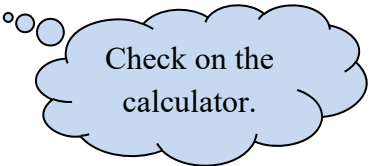
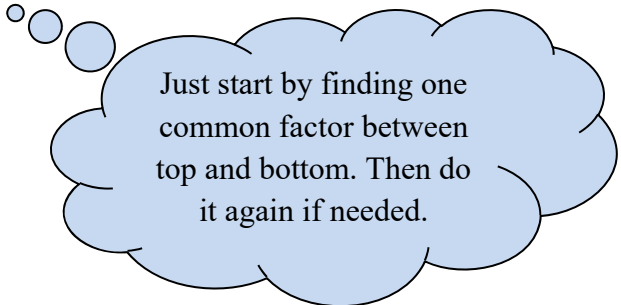
A fraction in **lowest terms** or in **simplified form** has no factors (other than 1) in common on the top and bottom. For instance, the fraction $\frac{5}{6}$ is in lowest terms but $\frac{4}{6}$ is *not*.

Find the factors of 4, 5, and 6 to show this to be true.



expl 8: Reduce / Simplify / Write in lowest terms.

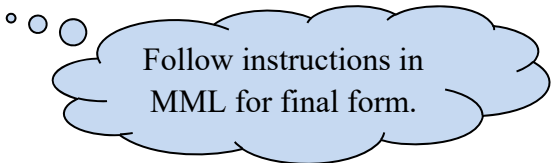
$$\frac{105}{90}$$



expl 9: Reduce / Simplify / Write in lowest terms.

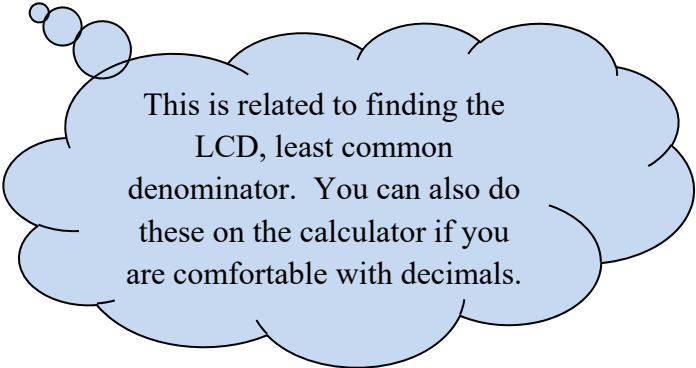
a.) $\frac{38}{24}$

b.) $\frac{28}{7}$

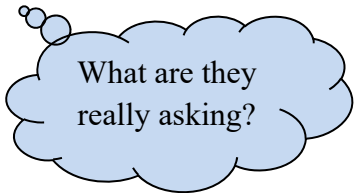


Comparing Fractions: Which is Larger?:

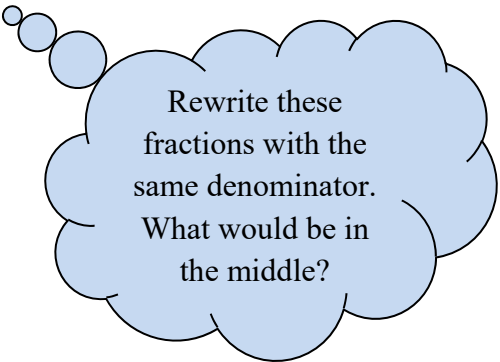
Which, $\frac{3}{5}$ or $\frac{4}{7}$, is larger? Turn both into fractions with the same denominator and then compare the tops.



expl 10: Is it possible to have a $\frac{7}{8}$ in. pipe with an inside diameter of $\frac{29}{32}$ inch?



expl 11: A $\frac{3}{4}$ in. drill bit is too large for a job but a $\frac{11}{16}$ in. drill bit is too small. What size should be tried next?



Worksheet: Understanding Fractions with Pictures:

This worksheet practices drawing out fractions as shown in the Notes. It explores equivalent fractions.