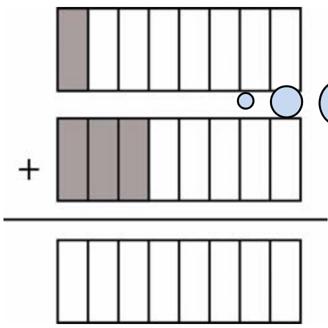
We have different procedures depending on if the bottoms are the same or different.

Technology-Integrated Mathematics
Class Notes
Fractions: Addition and Subtraction of Fractions (Section 2.4)

Adding and Subtracting Fractions with the Same Denominator (Bottom):
The book will call fractions that share a common denominator **like fractions**. Consider the addition problem $\frac{1}{8}+\frac{3}{8}$. Let’s look at a picture that will help us understand our procedure.



These “pieces” are all the same size. If we add 1 piece to 3 more, how many do we have now? Shade the picture under the line to illustrate this. What is the sum?

Always reduce! When we play with fractions, our final answers should be reduced. So, rewrite the answer in lowest terms.

expl 1: Add these fractions. Reduce to lowest terms.

a.) $\frac{1}{16}+\frac{3}{16}$

b.) $4/9+5/9+5/9+7/9$

If the answer will be an improper fraction, write it as a mixed number.

Subtracting Fractions with Like Denominators:
This works the same, except we subtract the top numbers (numerators).

expl 2: Subtract these fractions.

$^{13}\!/_{16} - {^{3}\!/_{16}}$

expl 3: Add and subtract as shown.

$\frac{4}{7}+\frac{5}{7}-\frac{3}{7}$

> Reduce to lowest terms even if the instructions do not say so.

Adding and Subtracting Mixed Numbers:
Some problems will start off with mixed numbers; there are two options to deal with these. First, add or subtract the whole parts and fractional parts separately, putting them together and simplifying in the end. Or second, we convert them to improper fractions and proceed as we did before.

First Method:

expl 4: Add $1\frac{5}{8}+3\frac{7}{8}$. We may write it in vertical format to begin.

$$\begin{array}{r} 1\frac{5}{8} \\ +3\frac{7}{8} \\ \hline \end{array}$$

Second Method:

expl 5: Add $3\frac{1}{4}+2\frac{3}{4}$. Here, let's convert both the improper fractions and then add.

expl 6: Use whichever method you like for the two problems here.

a.) $6\frac{1}{4}+4\frac{3}{4}$

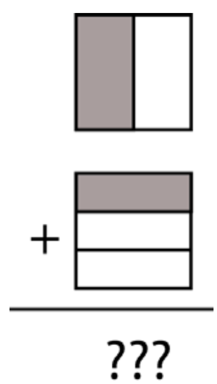
b.) $7\frac{4}{5}-3\frac{1}{5}$

Unlike Denominators:

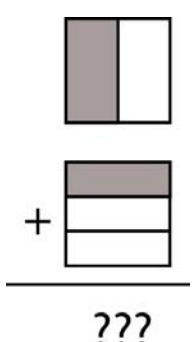
Consider the situation where we try to add two fractions that do *not* have the same denominator (called **unlike fractions**).

The picture illustrates the sum $\frac{1}{2}+\frac{1}{3}$. Do you see why?

Notice the "pieces" are *not* the same size and *cannot* just be added.



But wait! Below is the picture again. *Horizontally* divide the first square into three equal pieces. *Vertically* divide the second square into two equal pieces. Now, both squares are broken into six equal pieces and we can rewrite the fractions in terms of "sixths". Let's do that and then add.



Common Denominators and Equivalent Fractions:

We see that the fractions $^1/_2$ and $^1/_3$ could be written as $^3/_6$ and $^2/_6$, respectively.

Also, we could continue subdividing these little squares to see that, in fact,

$^1/_2 = {^3/_6} = {^6/_{12}} = {^{12}/_{24}} = \ldots$ and $^1/_3 = {^2/_6} = {^4/_{12}} = {^8/_{24}} = \ldots$

This string of fractions that represent the same number are called **equivalent fractions**. Our procedure for adding unlike fractions will be to convert each so that they have the same denominator (bottom). This is called a **common denominator**. Since smaller numbers usually make calculations easier, we will work toward getting the **least common denominator (LCD)**.

For the fractions we were playing with, $^1/_2$ and $^1/_3$, the LCD turned out to be 6.

Definition: **LCD** (**least common denominator**): The **LCD** is as described above. It turns out to be the least common *multiple* of the denominators we start with.

Let's practice finding multiples of whole numbers and see how that plays a role in what we do.

expl 7: Find the LCD of $\frac{1}{8}$ and $\frac{5}{12}$.

Write out the multiples of 8 and 12. Stop when you get one in common.

expl 8: Add.

$\frac{1}{8}+\frac{5}{12}$

Do it by hand and check on the calculator.

An Alternative Method to find the LCD:

Some people will prefer this method. We will use $\frac{1}{8}$ and $\frac{5}{12}$ as an example.

1. Factor both denominators using only prime numbers. $8 = ??$

$12 = ??$

called the **prime factorization**

2. The LCD will be the product of common factors (once) and all left-over factors.

expl 9: Try this method to find the LCD of $\frac{1}{4}$, $\frac{3}{10}$, and $\frac{11}{15}$.

Focus on the bottoms only.

For this example, this method may seem easier than the first method since writing the multiples of 4 all the way out to 60 would take a rather long time!

Mixed Numbers and LCD:

Let's try a few with mixed numbers. We will need our LCD knowledge.

expl 10: Add $2\frac{3}{8}+3\frac{1}{6}$.

expl 11: Subtract $4\frac{1}{8} - 1\frac{3}{4}$. Let's experiment with the two methods seen on page 2.

Method 1: Add or subtract the whole parts and fractional parts (with LCD in place) separately, putting them together and simplifying in the end.

For the first method from page 2, you'll need to "borrow".

Method 2: Convert them to improper fractions, get LCD, and proceed as we did before.

Can you check yourself on the calculator? Try it out by putting it all in at once.
If you put in 1 + ¾ for the second number, you'll need parentheses around it.

expl 12: A cabinet 30 inches high must have a $4\frac{1}{2}$-in. base and a $1\frac{3}{4}$-in. top. How much space is left for the drawers?