How do we find the area of a triangle, or better yet, an irregular shape?

Technology Integrated Mathematics
Class Notes
Geometry: Triangles, Regular Hexagons, and Irregular Polygons (Section 8.3)

Here, we explore and define triangles and hexagons. We will have many formulas concerning triangles including the Pythagorean Theorem. We will also look at irregular polygons like the floor of someone's garage with a small room tacked on one end.

Again, let's get started with some definitions.

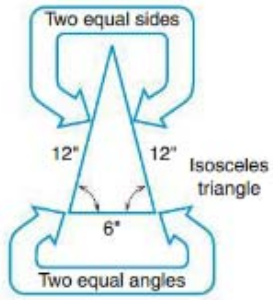
Recall: Definition: A **polygon** is a closed planar figure containing three or more angles and bounded by three or more straight lines. The word translates to "many sides".

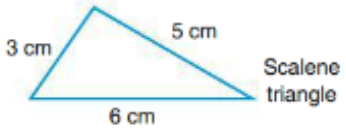
Definitions: Triangle: A **triangle** is a polygon with three sides and therefore three angles.

An **equilateral triangle** is one where *all* of the sides are of equal length. An **isosceles triangle** is one where *at least two* sides are equal in length. A **scalene triangle** is one in which *no* sides are equal in length.

Does that mean an equilateral triangle is also isosceles?

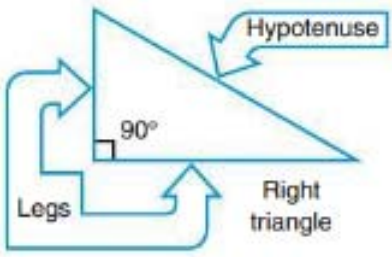
An **acute triangle** is one where *all* three angles are acute (less than 90°).
An **obtuse triangle** is one that contains *one* obtuse angle (greater than 90°).
A **right triangle** is one that contains a right angle (equal to 90°).





More about Right Triangles:
The sides that make up the right angle are called **legs**. The **hypotenuse** is the side opposite the right angle. The hypotenuse is always the longest side.

The Pythagorean Theorem will apply to right triangles only.

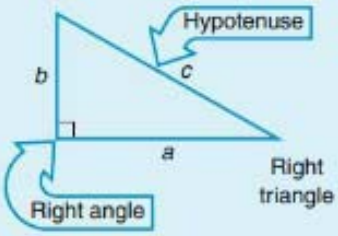


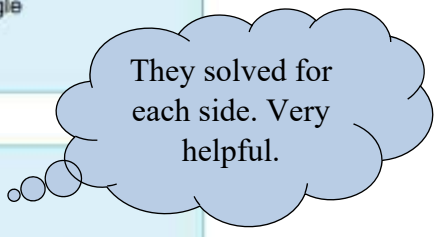
Pythagorean Theorem:
About 4,000 years ago, people started realizing a funny property of right triangles. It is named after a Greek philosopher named Pythagoras who lived around 2500 years ago. He founded a school and would not let his students eat beans because he thought farts let out a bit of your soul. Anyhoo…

Pythagorean Theorem

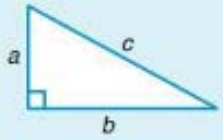
For any right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$c^2 = a^2 + b^2$



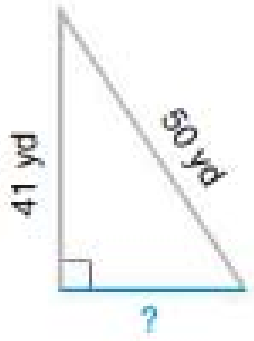


Equivalent Forms of the Pythagorean Theorem



$c = \sqrt{a^2 + b^2}$ $a = \sqrt{c^2 - b^2}$ $b = \sqrt{c^2 - a^2}$

expl 1: Find the missing side length. Include units. Round to the nearest tenth.



expl 2: Find the missing side length. Include units. Round to the nearest tenth.

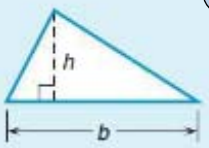
Finding the Area of a Triangle:

We have a few different formulas to help us find the area of a triangle. Our first one works for any triangle. Notice that you'll need to know the base and height of the triangle.

Area of a Triangle

Area, $A = \frac{1}{2}bh$ or $A = \frac{bh}{2}$

b = base h = height



A triangle can be thought of as half a parallelogram. Do you remember its area?

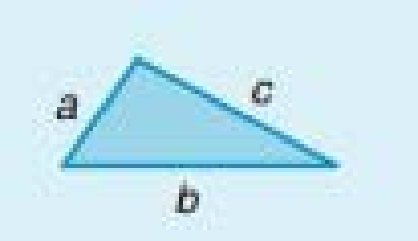
Above, h is the triangle's height. This is the *perpendicular* distance from the base (labeled b) to the third vertex.

expl 3: Find the area of this triangle. Include units (square feet).

Hero's Formula:
If you are only given the side lengths of a triangle, can you find its area? You betcha!

In fact, we have a very cool formula that was discovered almost 2000 years ago by the Greek mathematician Hero or Heron.

Here is a triangle with sides a, b, and c.



First, find half of the perimeter (or semiperimeter). We'll call this s. So, $s = \frac{a+b+c}{2}$.

Next, use the formula below to find the area.

$$A = \sqrt{s(s-a)(s-b)(s-c)}$$

expl 4: Find the area of this triangle. Include units (square feet). Round to the nearest tenth if needed.

Here are two more formulas. You can *only* use them, respectively, for isosceles and equilateral triangles. It is up to you if you use them at all.

Area of an Isosceles Triangle

Area, $A = \frac{1}{2}b\sqrt{a^2 - \left(\frac{b}{2}\right)^2}$

Area of an Equilateral Triangle

Area, $A = \frac{a^2\sqrt{3}}{4} \approx 0.433a^2$

Regular Polygons, Pentagons, and Hexagons:
Definition: Regular polygon: A **regular polygon** is one which has equal side lengths. Equilateral triangles and squares are regular polygons.

Definition: Regular pentagon: A **regular pentagon** is a polygon with five sides, all of equal length. Its interior angles are also all equal.

Definition: Regular hexagon: A **regular hexagon** is a polygon with six sides, all of equal length. Its interior angles are also all equal.

We have the formula for the area of a regular hexagon. Also, we know the relationship between the dimensions of a regular hexagon.

Area of a Regular Hexagon

Area, $A = \frac{3a^2\sqrt{3}}{2} \approx 2.598a^2$

Carefully look at what they are measuring.

Dimensions of a Regular Hexagon

Distance across the corners, $d = 2a$ or $a = 0.5d$

Distance across the flats, $f \approx 1.732a$ or $a \approx 0.577f$

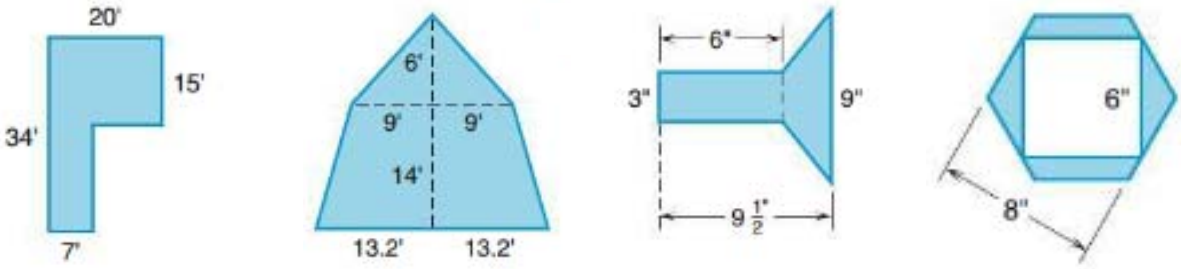
expl 5a: Find the missing dimension for this regular hexagon. Round to the nearest tenth if needed and include units.

expl 5b: Find the area of the regular hexagon pictured above. Round to the nearest tenth if needed and include units.

expl 6: Find the missing dimension for this regular hexagon. Round to the nearest tenth if needed and include units.

Irregular Polygons:

Irregular polygons will be made up of assorted polygons stuck together. Here are some examples. Notice how you can break them up into triangles, rectangles, trapezoids, etc.



expl 7: How many square feet of drywall is needed for this wall?