Units are what gives our numbers context and meaning. We will also work with proper rounding.

Technology Integrated Mathematics
Class Notes
Measurement: Working with Measurement Numbers (Section 5.1)

Most numbers we use measure something. Our numbers will most often have **units** like feet, meters, miles per hour, tons, or pound-feet. It is crucial that we consider units in our calculations and record the units of any answer we give.

A Cautionary Tale:
On December 11, 1998, NASA launched a $125 million robotic space probe called the Mars Climate Orbiter. On its approach to Mars in September 1999, the space probe dramatically crashed through the atmosphere of Mars and was crippled. Why? Many companies were supplying various systems for the probe. It turns out that Jet Propulsion Laboratory (JPL) was using metric units (millimeters and meters) while a subcontractor, Lockheed Martin Astronautics, was using English units (inches, feet, and pounds). Oops. Units matter.

Significant Digits:
Our measurement tools have limitations that determine how accurately they will measure. We must often round the numbers using procedures we have discussed. **Significant digits** in a number are those that represent an actual measurement. There are standard rules for significant digits we will follow.

Rule 1: Digits other than zero (0) are always significant.

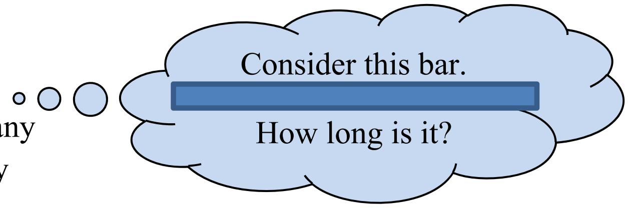
Rule 2: A zero (0) is significant when it
(a) appears between two significant digits,
(b) is at the right end of a decimal number like 24.0 or 23.40, or
(c) is marked as significant with an overbar.

Rule 3: A zero (0) is *not* significant when it
(a) is at the right end of a whole number, or
(b) is at the left end of a number.

expl 1: Determine the number of significant digits according to the rules given.

A tachometer reading of 93.2 rpm has ________ significant digits. (Rule 1)
A pressure reading of 2706 torr has ________ significant digits. (Rule 2a)
A length measurement of 22.50 feet has ________ significant digits. (Rule 2b)
A voltage reading of $24\bar{0}$ volts has ________ significant digits. (Rule 2c)
A voltage reading of 230 volts has ________ significant digits. (Rule 3a)
A current reading of 0.035 amp has ________ significant digits. (Rule 3b)

Precision and Accuracy:
We could measure the bar on the right with many different tools. If we used a ruler, we might say that it is 2 inches long.



However, if we used a better measuring device, we might measure it as 1.9 inches. Grab a set of calipers and we might end up with 1.9375 inches. We could even imagine ourselves whipping out a microscope and really getting in there. Perhaps, we would say the bar is 1.9375235 inches long. We are getting *more and more precise* with each step.

Definitions: Precision and Accuracy: The **precision** of a measurement number is indicated by the place value of its *right-most significant digit*. The **accuracy** of a measurement number refers to the *number of significant digits* that it contains.

expl 2: State the precision and accuracy of each measurement.
a.) 5.23 seconds

Precision will be stated as the place value (hundredth, thousandth, etc.) or simply 0.01, 0.001, etc. of this right-most significant digit. **Accuracy** will be stated to a number of significant digits.

b.) 50 minutes

c.) 56,000 feet

d.) 56,213 feet

Greatest Possible Error:

The **greatest possible error** of a measurement is defined to be half of the precision of the measurement. If you are reading a tool, the **greatest possible error** is half of the smallest division on the scale of the tool.

A tachometer shows an engine speed of 4700 rpm. The precision of this instrument is 100 rpm. So, the greatest possible error is half that, or 50 rpm. We conclude the actual speed of the engine is 4700 ± 50 or somewhere between 4650 and 4750 rpm.

expl 3: State the greatest possible error of each measurement.

a.) 5.23 seconds

Halve the precision.

b.) 50 minutes

c.) 56,000 feet

d.) 56,213 feet

Tolerance:
We saw tolerance in an earlier section. You may see it here as it relates to precision. If you are told that the dimension on a machine tool is given as 2.835 ± 0.004 in., that means the dimension should be 2.835 inches to within 4 thousandths of an inch. In other words, the dimension should measure between 2.831 and 2.839 inches to be tolerated (or allowed).

Addition and Subtraction of Measurement Numbers:
When we add or subtract numbers, we do *not* want to imply a greater precision than the numbers we started with. We will always round a sum or difference to agree in precision with the *least precise* measurement number involved.

The units of these problems will carry over. For example, if we add inches to inches, we get inches.

expl 4: Perform the operation and round to the appropriate number of significant digits.
a.) 5.25 in. + 0.3 in. + 0.50 in.

b.) 1.565 oz. – 0.38 oz.

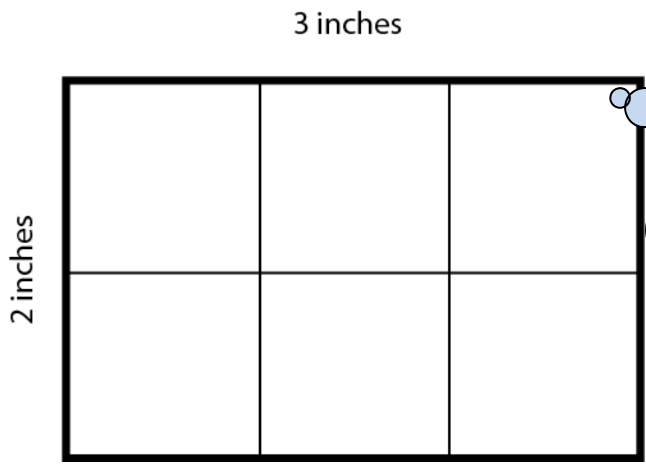
c.) 3.2 lb + ¼ lb

Convert all to decimal numbers. Convention says we round to agree with the precision of the *least precise decimal* measures.

Multiplication and Division of Measurement Numbers:
Round these quotients and products to the same number of significant digits as the *least accurate* number used in the calculation.

Units do *not* work the same as when we add or subtract. Let's investigate that.

Consider the product 2 in. x 3 in. We know that 2 times 3 makes 6, but what units would we have now? We can visualize this product as the area of a rectangle. One side of this rectangle is 2 inches long and the other side is 3 inches long. Do you see that this rectangle is comprised of 6 squares that are all 1 inch by 1 inch?



> These individual squares are "square inches". Hence, we say this rectangle has an area of 6 square inches. You can abbreviate it as sq in. or in^2.

Definition: Compound units: Units that are a combination of simpler units. Examples are square inches, miles per hour, pound-feet, rpm, and feet per second (ft/sec).

A good way to see how units come together to make compound units is to use unit analysis as shown earlier. For the picture above, we can show

2 inches x 3 inches
= (2 x 3) x (1 inch x 1 inch)
= 6 square inches

> To be clear, we are *not* really multiplying these *words*. It is simply a good way to keep the units straight.

Consider a car travelling 50 miles per hour for 2 hours. The distance they travel is

$$50\,\frac{miles}{hour}\times 2\ hours$$
$$=50\times 2\left(\frac{miles}{hour}\times\frac{hours}{1}\right)$$
$$=100\ miles$$

> Our formula is $d = rt$ where d is distance, r is rate, and t is time.

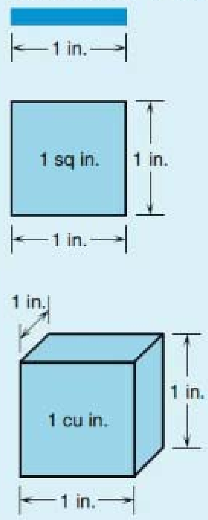
Length Versus Area Versus Volume:

We might use inches to measure length and square inches to measure area.

What do we use to measure volume? Here is a handy picture from the book.

Visualizing Units

It is helpful to have a visual understanding of measurement units and the "dimension" of a measurement.



The length unit **inch,** in., specifies a **one**-dimensional or linear measurement. It gives the length of a straight line.

The area unit **square inch,** sq in., specifies a **two**-dimensional measurement. Think of a square inch as giving the area of a square whose sides are one inch in length.

The volume unit **cubic inch,** cu in., specifies a **three**-dimensional measurement. Think of a cubic inch as the volume of a cube whose edges are each one inch in length.

Let's look at some examples. Again, we will round these quotients and products to the same number of significant digits as the *least accurate* number used in the calculation.

Unlike we when added or subtracted numbers, we use *accuracy* (*not precision*) to determine how to round.

expl 5: Multiply or divide as shown. Round to the appropriate number of significant digits.

a.) 6 ft x 7 ft

b.) 458 miles ÷ 7.3 hours

c.) $65 ÷ 3 lb

Do *not* forget units!

An Exception: When Using Counts:
An exception to what we just did occurs when we are using a number obtained from simple counting. These numbers are considered **exact** and do *not* affect the accuracy of the calculation. For example, if a flywheel is known to have turned 48 revolutions in 1.25 minutes, we will find the revolutions per minute as 48/1.25 or 38.4 rpm. Since the 48 number is considered exact, we ignore the fact that it has an accuracy of 2 significant digits. Instead, we will take the 1.25 to be the "least accurate *measurement* number". Hence, we want 3 significant digits in our answer. We will end up with 38.4 rpm as our final answer.

Conversion Between Decimal and Fraction Forms:
Often, a technical drawing will use decimal numbers while the ruler you use has fractional divisions. When we convert, we will inevitably have some error and we need to be able to calculate the error.

expl 6: Find the closest fractional equivalent for the following decimal measurements using the given denominator in parentheses. Then find the error, to the nearest ten-thousandth (4 decimal places), if the fraction is used in place of the decimal number.
a.) 0.921 in. (16ths)

How many "sixteenths" is closest to 0.921? Multiply by 16/16 and round the top to the nearest whole number.

To find the error, subtract the given decimal from your fraction.

b.) 2.091 in. (32nds)