A fan's blades rotate fully in 0.125 seconds. What is the linear speed of the blade tip?

Technology Integrated Mathematics
Class Notes
Trigonometry: Angles and Right Triangles (Section 10.1)

We have already learned some basic information about angles and how they are measured. Here, we get into more detail. We will see radian measure which is an alternative to degrees. We will see how degrees can be broken into smaller pieces called seconds and minutes. We will continue exploring right triangles.

**Definitions**: **Seconds** and **Minutes**: A single degree can be broken into 60 equal pieces called **minutes**. A minute can be further broken down into 60 equal pieces called **seconds**. The book gives us these conversion factors and some helpful abbreviations.

60 minutes = 1 degree   abbreviated, $60' = 1°$
60 seconds = 1 minute   abbreviated, $60'' = 1'$

Recall that a full circle is 360°. A right angle is 90°.

For most technical purposes, we round angles to the nearest minute. In the trades, we will usually round to the nearest degree.

expl 1: Write the following in terms of degrees and minutes.
a.) 46.55°

Convert the fractional part to minutes.

b.) 39 ¼ °

**Converting Angle Units Using a Calculator**

Scientific calculators have special keys designed for entering angles in degrees and minutes and for performing angle conversions. There is normally a key marked [° ′ ″] that is used for entering and converting angles. You may instead see a key marked [▸DMS] or [▸DD] to be used for this purpose. Because the procedures for doing these conversions vary greatly from model to model, we leave it to each student to learn the key sequences from his or her calculator's instruction manual.

expl 2: Convert to decimal degree measure. Round to the nearest hundredth if needed.
$45°39'$

**Radian Measure:**

A full circle measures 360°. Radians are also used to measure angles and a full circle is said to be $2\pi$ radians. We have these helpful conversion factors.

$$1 \text{ radian} = \frac{180°}{\pi} \text{ or about } 57.296°$$

So, $2\pi$ radians is about 6.3 radians.

$$\text{Radians} = \text{degrees} \cdot \frac{\pi}{180°}$$

$$\text{Degrees} = \text{radians} \cdot \frac{180°}{\pi}$$

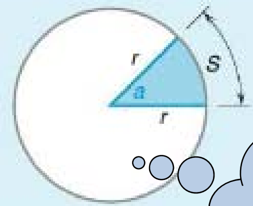
expl 3: Write the angle $64\frac{1}{2}°$ in radians. Use the $\pi$ button on the calculator; do *not* round $\pi$ to 3.14. Round the final answer to the nearest hundredth. Include "radians" or "rad" as the units.

**Sectors:**
**Definition: Sector:** a wedge-shaped portion of a circle taken from the center out toward the circle itself.

The book gives us these formulas. We will use radian measures.

### Sectors

| | | |
|---|---|---|
| Arc length | $S = ra$ | |
| Area | $A = \frac{1}{2}r^2a$ | where $a$ is the central angle given in radians. |




The angle must be in radians!

expl 4: Find the area $A$ and arc length $S$ of the following sector. Draw a picture of the arc.
$a$ = 1.2 radians
$r$ = 130 feet

**Linear and Angular Speed:**
Radians are used to describe the rotation of an object around a single central point. As the object travels in this circle, we can find its **angular speed**, just as we would find the **linear speed** of a car travelling along a straight line.

**Linear Speed:**

An object travels a distance $d$ in time $t$. Its **average linear speed** $v$ is $v = \frac{d}{t}$. The units we use is a compound unit found by dividing the distance units by the time units.

**Example:** A car travels 50 miles in two hours. The car's average linear speed is 50/2 or 25 miles per hour.

We have seen this formula as $d = r \cdot t$ before.

**Angular Speed:**
An object travels along a circular arc defined by an angle measured as $a$ radians in time $t$. **Its average angular speed** is $w = \frac{a}{t}$. The units we use is a compound unit found by dividing radians by the time units.

**Example:** A flywheel rotates through an angle of 2.50 radians (about 40% of a full circle) in 1.4 seconds. The flywheel's average angular speed is 2.50/1.4 or about 1.8 radians per second.

expl 5: The blades of a fan have a radius of 16.5 inches and they turn one revolution every 0.125 seconds.
a.) Calculate the *linear* speed of the tip of a blade. Round to the nearest whole number.

What distance does the blade tip travel in 0.125 seconds?

b.) Calculate the average *angular* speed of the fan blade. Round to two decimal places.

One revolution is $2\pi$ radians.

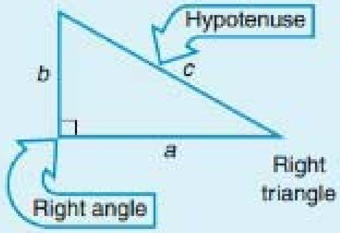
**Right Triangles:**
We will move on from here to study trigonometry of right triangles. Recall that a **right triangle** is one which contains one right angle (90°). The right angle should be marked by a little square corner.

Recall we have the **Pythagorean Theorem** (copied below) to help us find missing sides of right triangles. You will practice that a bit in the homework but we will focus here on other truths about some special triangles.

### Pythagorean Theorem

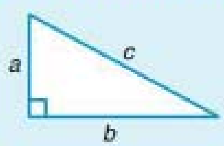
For any right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

$c^2 = a^2 + b^2$




They solved for each side. Very helpful.

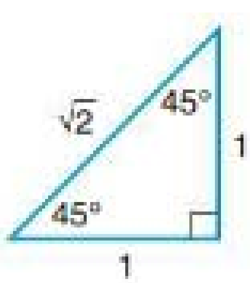
### Equivalent Forms of the Pythagorean Theorem

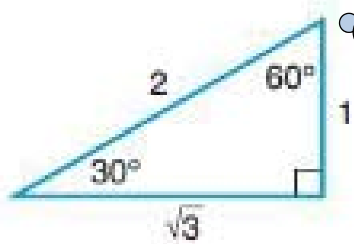



$c = \sqrt{a^2 + b^2}$ $a = \sqrt{c^2 - b^2}$ $b = \sqrt{c^2 - a^2}$

**Special Right Triangles:**

We will see these right triangles a lot. We call them the 45°-45°-90° right triangle and the 30°-60°-90° right triangle. There are very specific relationships among the side lengths.




The sides maintain these ratios among themselves. Let's see how we can use that.

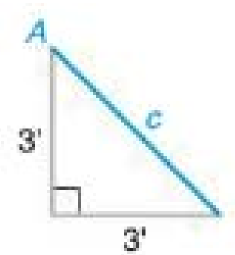
expl 6: Find the quantities indicated using the known ratios among sides. Round sides to the nearest tenth. Include units.


a.) angle $T =$

b.) side length $t =$

c.) side length $e =$

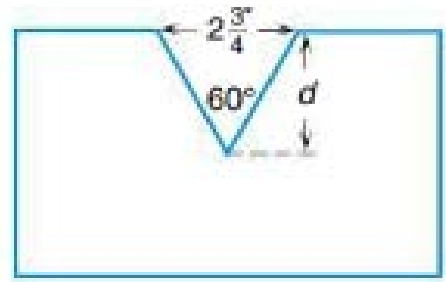
expl 7: Find the quantities indicated using the known ratios among sides. Round sides to the nearest tenth. Include units.




a.) angle $A =$

b.) side length $c =$

expl 8: Find the depth $d$ of the $V$-slot shown here. Round to the nearest hundredth of an inch.




Redraw *half* the V-slot as a 30°-60°-90° right triangle