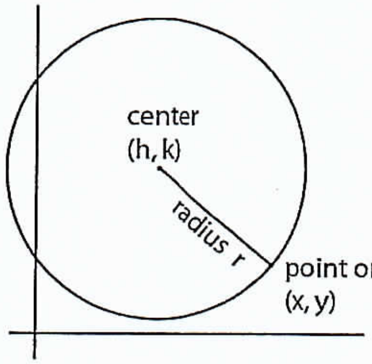
A circle is a collection of points that are all the same distance from another single point.

College algebra
Circles (Section 2.4)

Here, we play with circles plotted on an xy-plane. We can use algebra to define and examine the circle using an equation in x and y. First, the basics.

Equations of Circles:
A circle is the set of all points (x, y) equidistant from a single point (h, k) called the **center.**



Using r for the **radius** (the distance between the center and any point on the circle) and the distance formula, we get the equation $r = \sqrt{(x-h)^2 + (y-k)^2}$. Square both sides and you get the following formula.

Standard Form of the Equation of a Circle:
The **equation of a circle** is given as $(x-h)^2 + (y-k)^2 = r^2$. The point (h, k) is the center and r is the radius.

Can you write this equation using (0, 0) as the center?

$(h,k) = (0,0)$
$(x-0)^2 + (y-0)^2 = r^2$
$x^2 + y^2 = r^2$

expl 1: The unit circle is centered at (0, 0) and has a radius of 1 unit. What would its equation look like?

$x^2 + y^2 = 1^2$

$x^2 + y^2 = 1$

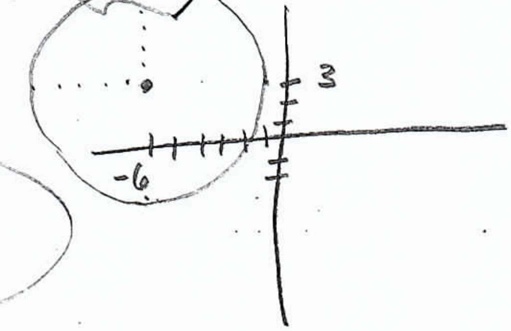
expl 2: Find the center (labeled as (h, k)) and radius (labeled as r) of the circle below. Then draw a quick xy-plane and graph the circle on paper.

$(x+6)^2 + (y-3)^2 = 25$ ← $r = 5$ because $5^2 = 25$

$k = 3$

$h = -6$

$x+6 = x - -6$ (h)



$(h,k) = (-6,3)$ center
$r = 5$ radius

expl 3: Find the center and radius of the circle below. Then draw a quick *xy*-plane and graph the circle on paper.

$3(x-3)^2+3(y+2)^2=48$

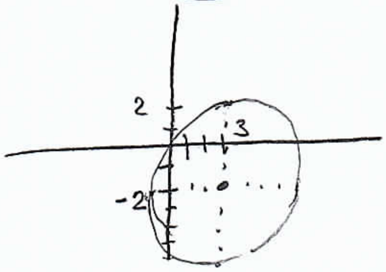
How is this equation different from the standard form above?

$$\frac{3(x-3)^2}{3} + \frac{3(y+2)^2}{3} = \frac{48}{3}$$

$$(x-3)^2 + (y+2)^2 = 16$$

$h=3 \qquad k=-2 \qquad r=4$

Center $(3,-2)$ radius $=4$



expl 4: Find the equation of a circle that has a center of $(6, -5)$ and passes through the point $(1, 7)$.

The point (1, 7) is *on* the circle.

The equation of a circle is $(x-h)^2+(y-k)^2=r^2$. Put in what you know.

$$(x-h)^2+(y-k)^2=r^2$$

$$(x-6)^2+(y+5)^2=r^2$$

$$(1-6)^2+(7+5)^2=r^2$$

$$(-5)^2+12^2=r^2$$

$$25+144=r^2$$

$$169=r^2 \rightarrow r=13$$

Eqn: $\boxed{(x-6)^2+(y+5)^2=169}$

Another way to express the relationship.

General Form of the Equation of a Circle:

The **equation of a circle** can also be given as $x^2+y^2+ax+by+c=0$.

To go from the Standard form to the General form of a circle, you merely have to FOIL and rearrange some terms. It takes completing the square to go the other way. Let's go over both of these types of problems.

Converting from Standard to General Form:

expl 5: Convert this equation to the General Form for this circle.

$(x+6)^2+(y-3)^2=25$

$$(x+6)(x+6)+(y-3)(y-3)=25$$

$$x^2+6x+6x+36+y^2-3y-3y+9=25$$

$$x^2+12x+36+y^2-6y+9=25$$

$$x^2+y^2+12x-6y+20=0$$

Completing the Square:
Completing the square is a technique that forces an expression in the form $x^2 + ?x$ into the form $(x + ??)^2$. Let's look at why completing the square works.

Look at this pattern: FOIL these problems.

$(x+4)^2 = x^2 + \underline{\underline{8x}} + \underline{\underline{16}}$

$(x+7)^2 = x^2 + \underline{\underline{14x}} + \underline{\underline{49}}$

$(x-5)^2 = x^2 - \underline{\underline{10x}} + \underline{\underline{25}}$

Recall these are perfect square trinomials.

So, we are interested in going from $x^2 + 8x + 16$ back to the $(x + 4)^2$ form. But what if we were just given $x^2 + 8x$? How would we figure out the constant that "completed" $x^2 + 8x$ so that we could factor it as $(x + 4)^2$?

In each trinomial above, what is the relationship between the coefficient of the x-term and the constant at the end? coeff of x-term squared makes constant at end.

What would you add to $x^2 + 12x$ so that we could write it as $(x + ?)^2$, and what goes in the parentheses? $x^2 + \underline{\underline{12x}} + 6^2 \rightarrow x^2 + 12x + 36 = (x+6)^2$

Now that we have the general idea of completing the square, let's use it to convert the equation of a circle to Standard Form. Do *not* lose sight of the fact that this is an equation whose left and right sides must be equal at all times.

Converting from General to Standard Form:
expl 6: Convert this equation to the Standard Form for this circle. Then determine the center, radius, and intercepts.

$x^2 + y^2 + 4x + 2y - 20 = 0$

Write it as
$x^2 + 4x \qquad + y^2 + 2y \qquad = 20$
and complete each square. Do *not* forget about the right side!

$x^2 + \underline{\underline{4x}} \qquad + y^2 + \underline{2y} \qquad = 20$

$\frac{1}{2} \cdot 4 = 2 \qquad \frac{1}{2} \cdot 2 = 1$

$2^2 = 4 \qquad 1^2 = 1$

$x^2 + 4x \underline{\underline{+4}} + y^2 + 2y \underline{\underline{+1}} = 20 + 4 + 1$

$(x+2)^2 + (y+1)^2 = 25$

$\boxed{\begin{array}{l}\text{center } (-2, -1) \\ \text{radius } r = 5\end{array}}$

x-int: $(y = 0)$

$(x+2)^2 + (0+1)^2 = 25$

$(x+2)^2 = 24$

$\sqrt{(x+2)^2} = \sqrt{24}$

$x + 2 = \pm\sqrt{24}$

y-int: $(x = 0)$

$(0+2)^2 + (y+1)^2 = 25$

$4 + (y+1)^2 = 25$

$(y+1)^2 = 21$

$\sqrt{(y+1)^2} = \sqrt{21}$

$y + 1 = \pm\sqrt{21}$

y-int: $y = -1 \pm \sqrt{21}$

Graphing on the Calculator:
Graphing an equation in x and y involves solving for y. Remember that this will introduce a "plus or minus" into our solution. Let's try one out.

expl 7: Graph the circle on your calculator. Does your graph look like your picture from page 1?
$(x+6)^2+(y-3)^2=25$

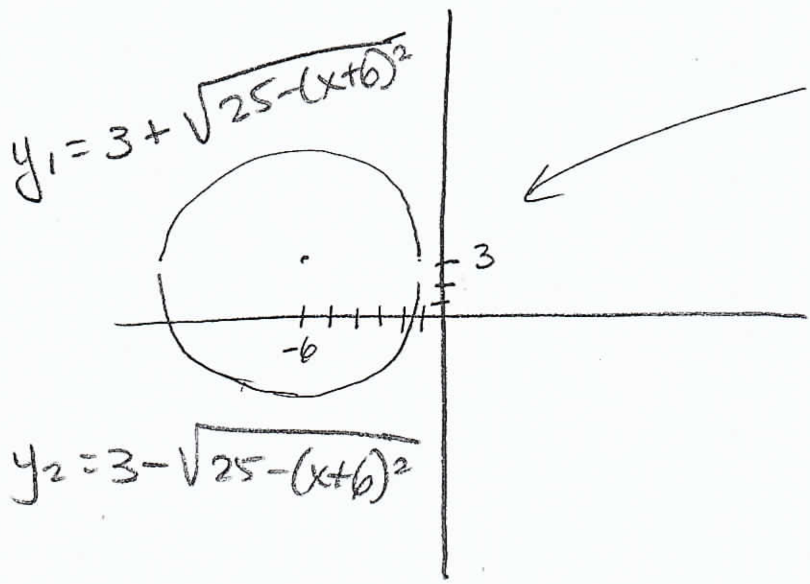
Selecting Zoom: Square will help your circle look more circular.

$$(y-3)^2 = 25-(x+6)^2$$

$$\sqrt{(y-3)^2} = \sqrt{25-(x+6)^2}$$

$$y-3 = \pm\sqrt{25-(x+6)^2}$$

$$y = 3\pm\sqrt{25-(x+6)^2}$$



does not look to connect because of pixels on Calculator.