College algebra
Direct, Inverse, and Joint Variation
Section 2.5

If two variables are related to each other by a constant ratio or product, we say we have a direct or inverse variation.

Direct Variation:
We will start with an example.

Margie gets paid \$10 per hour when she babysits. Fill in the table for the various number of hours given.

x, number of hours	*y*, total charge	Find *y/x*
1	\$ 10	\$10
2	\$ 20	\$10
3	\$ 30	\$10
7	\$70	\$10

The ratio is constant which means there is a direct variation here.

Definition: Direct Variation:
If a situation can be modeled by the linear function $f(x) = kx$, or $y = kx$, where k is a nonzero constant, we say that it is a **direct variation**. We could say y **varies directly as** x **or** y **is directly proportional to** x. The number k is the **variation constant** or the **constant of proportionality**.

Can you think of any other variables that would be directly proportional?

? . . .

Inverse Variation:

Again, let's look at an example. The area of a rectangle is 90 square meters. Fill in the table for the various widths given.

x, width	y, length	Find $x \cdot y$
5	18	90
10	9	90
15	6	90
30	3	90

The product is constant which means there is an inverse variation here.

Definition: Inverse Variation:

If a situation can be modeled by the linear function $f(x) = k/x$, or $y = k/x$, where k is a nonzero constant, we say that it is an **inverse variation**. We could say y **varies inversely as** x or y **is inversely proportional to** x. The number k is the variation constant or the constant of proportionality.

Can you think of any other variables that would be inversely proportional?

? ...

expl 1: Find the variation constant and the equation of variation for the given situation.

a.) y varies directly as x, and $y = 54$ when $x = 12$

$y = kx$

$y = kx$

$54 = k(12)$

$4.5 = k \rightarrow \boxed{y = 4.5x}$

b.) y varies inversely as x, and $y = 12$ when $x = 5$

$y = k/x$

$y = k/x$

$12 = k/5$

$60 = k \rightarrow \boxed{y = 60/x}$

W = weight a beam can support
L = length of beam

expl 2: The weight W that a horizontal beam can support varies inversely as the length L of the beam. Suppose an 8 meter beam can support 1200 kg. How many kilograms can a 14 meter beam support?

$W = K/L$ for some $k \in \mathbb{R}$

$y = kx$ OR $y = k/x$?

$1200 = k/8$

$9600 = k$

Find k and then form the equation of variation.

$W = 9600/L$

$W = 9600/14$

$W \approx 685.7$ kg

expl 3: The ~~relative aperture~~, or f-stop, of ~~a 23.5 mm diameter~~ camera lens is directly proportional to the focal length F of the lens. If a focal length of 150 mm has an f-stop of 6.3, find the f-stop of this lens with a focal length of 80 mm.

f = f-stop

F = focal length

$y = kx$ OR $y = k/x$?

$f = k \cdot F$

Find k and then form the equation of variation.

$6.3 = k \cdot 150$

$K = 0.042$

$f = 0.042\,F$

$f = 0.042(80)$

$\boxed{f = 3.36 \text{ f-stop}}$

Combined Variation:

There are three related types of variation we will study.

1. y **varies directly as the** n**th power of** x if there is some nonzero constant k such that

$$y = kx^n.$$

2. y **varies inversely as the** n**th power of** x if there is some nonzero constant k such that

$$y = \frac{k}{x^n}.$$

3. y **varies jointly as** x **and** z if there is some nonzero constant k such that $y = kxz$.

General Procedure:

1. Read what type of variation (direct, inverse, joint, or some combination) the variables have in the problem.
2. Start with the general equation given above. Define your variables!
3. Use the given information to find the missing proportionality constant k.
4. Rewrite the equation with your value of k in place. Use it to answer the question.

expl 4: The weight W of an object varies inversely as the square of the distance d from the center of the earth. At sea level (3978 miles from the center of the earth), an astronaut weighs 220 pounds. Find his weight when he is 200 miles above the surface of the earth.

$$W = \frac{k}{d^2}$$

What is d when he is 200 miles above the earth?

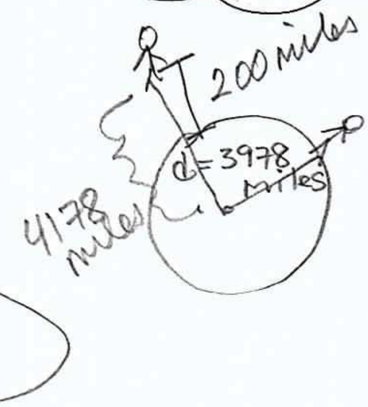
$W = k/d^2$ for some $k \in \mathbb{R}$

$220 = k/3978^2$

$k = 3{,}481{,}386{,}480$

$$W = \frac{3{,}481{,}386{,}480}{d^2}$$

$$W = \frac{3481386480}{4178^2}$$



$\boxed{W \approx 199.44 \text{ pounds}}$

$k = ?$

expl 5: Find the variation constant and the equation of variation for the given situation.

y varies jointly as x and z and inversely as w, and $y = 14$ when $x = 3$, $z = 2$, and $w = 2$

$$y = \frac{k \cdot xz}{w} \quad \text{for some } k \in \mathbb{R}$$

$$14 = \frac{k \cdot 3 \cdot \cancel{2}}{\cancel{2}}$$

$$14/3 = k$$

$$y = \frac{14/3 \cdot x \cdot z}{w}$$

$$\boxed{y = \frac{14xz}{3w}}$$

expl 6: The kinetic energy K of a moving object varies jointly with its mass m and the square of its velocity v. An object with a mass of 25 kilograms that is moving with a velocity of 10 meters per second has a kinetic energy of 1250 joules. If this same object is now moving at 35 meters per second, what is its kinetic energy?

First, we find k, the proportionality constant. Do *not* confuse this with K, the kinetic energy.

K = kinetic energy of moving object.

m = mass (object)

v = velocity (object)

$$K = k \cdot m \cdot v^2 \qquad k \in \mathbb{R}$$

$$1250 = k \cdot 25 \cdot 10^2$$

$$1250 = k \cdot 2500$$

$$1/2 = k$$

$$K = \frac{1}{2} m v^2$$

$$= \frac{1}{2}(25)(35)^2$$

$$K = 15312.5 \text{ joules}$$