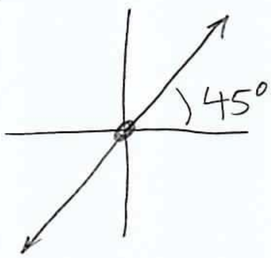
Do you remember these functions? We will use them as the base for other functions.

College algebra
Library of Functions and Piecewise Functions
Section 3.4

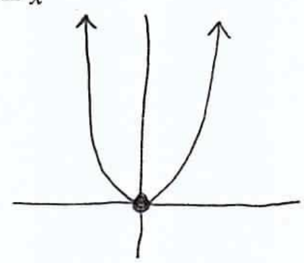
Library of Functions:

Draw from memory or use your calculator (on the Standard window) to graph the following functions. You should acquaint yourself with their basic shapes.

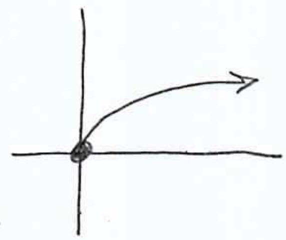
Identity function
$y = x$



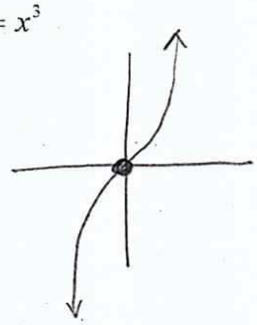
Square function
$y = x^2$



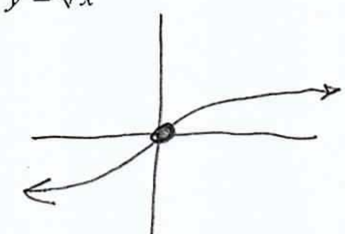
Square root function
$y = \sqrt{x}$



Cube function
$y = x^3$



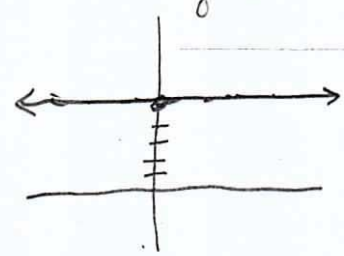
Cube root function
$y = \sqrt[3]{x}$



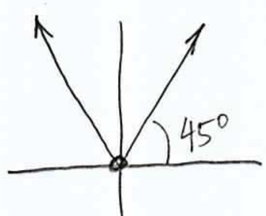
Constant function
$y = b$, b is a real number

Make up b.

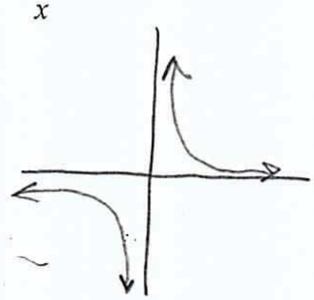
$y = 5$



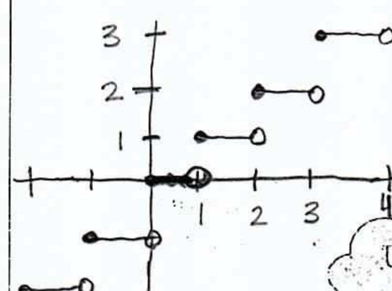
Absolute value function
$y = |x|$



Reciprocal function
$y = \frac{1}{x}$



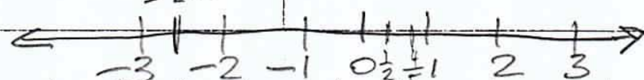
Greatest integer function
$y = int(x)$ = greatest integer less than or equal to x



Use tick marks.

$y = int(x)$

x	$y = int(x)$
0	0
1/2	0
4/5	0
1	1
1.5	1
2.3	2
2.7	2
3	3
-2.5	-3
-3	-3



Where are these functions increasing, decreasing, constant? Where are their (x and y) intercepts? Later, we will study how to transform these graphs by shifting, reflecting, stretching, and shrinking (also called compressing or squashing) the graphs.

Integers: $\{\ldots, -3, -2, -1, 0, 1, 2, 3, \ldots\}$

Properties of Base Functions:

For each of the functions above, we will investigate several questions. Consult the information below. (I abbreviated increasing/decreasing/constant as inc/dec/cnst.)

Identity function $y = x$ domain: $(-\infty, \infty)$ range: $(-\infty, \infty)$ x-intercept(s): $x = 0$ y-intercept: $y = 0$ even or odd?: odd inc/dec/cnst?: inc: $(-\infty, \infty)$ mins/maxes: none	**Square function** $y = x^2$ domain: $(-\infty, \infty)$ range: $[0, \infty)$ x-intercept(s): $x = 0$ y-intercept: $y = 0$ even or odd?: even inc/dec/cnst?: dec: $(-\infty, 0]$ inc: $[0, \infty)$ mins/maxes: abs. min. of $y = 0$ at $x = 0$	**Square root function** $y = \sqrt{x}$ domain: $[0, \infty)$ range: $[0, \infty)$ x-intercept(s): $x = 0$ y-intercept: $y = 0$ even or odd?: neither inc/dec/cnst?: inc: $[0, \infty)$ mins/maxes: abs. min. of $y = 0$ at $x = 0$
Cube function $y = x^3$ domain: $(-\infty, \infty)$ range: $(-\infty, \infty)$ x-intercept(s): $x = 0$ y-intercept: $y = 0$ even or odd?: odd inc/dec/cnst?: inc: $(-\infty, \infty)$ mins/maxes: none	**Cube root function** $y = \sqrt[3]{x}$ domain: $(-\infty, \infty)$ range: $(-\infty, \infty)$ x-intercept(s): $x = 0$ y-intercept: $y = 0$ even or odd?: odd inc/dec/cnst?: inc: $(-\infty, \infty)$ mins/maxes: none	**Constant function** $y = b$, b is a real number domain: $(-\infty, \infty)$ range: $\{b\}$ x-intercept(s): none unless $b = 0$ y-intercept: $y = b$ even or odd?: even inc/dec/cnst?: cnst: $(-\infty, \infty)$ mins/maxes: abs. min. and abs. max. of $y = b$ for all x
Absolute value function $y = \|x\|$ domain: $(-\infty, \infty)$ range: $[0, \infty)$ x-intercept(s): $x = 0$ y-intercept: $y = 0$ even or odd?: even inc/dec/cnst?: dec: $(-\infty, 0]$ inc: $[0, \infty)$ mins/maxes: abs. min. of $y = 0$ at $x = 0$	**Reciprocal function** $y = \dfrac{1}{x}$ domain: $(-\infty, 0) \cup (0, \infty)$ range: $(-\infty, 0) \cup (0, \infty)$ x-intercept(s): none y-intercept: none even or odd?: odd inc/dec/cnst?: dec: $(-\infty, 0) \cup (0, \infty)$ mins/maxes: none	**Greatest integer function** $y = int(x)$ = greatest integer less than or equal to x domain: $(-\infty, \infty)$ range: $\{y \mid y \text{ is an integer}\}$ x-intercept(s): $0 \le x < 1$ y-intercept: $y = 0$ even or odd?: neither inc/dec/cnst?: cnst: every interval of the form $[k, k+1)$ for k an integer mins/maxes: none

expl 1: For the function $f(x) = int(3x)$, find the following.

a.) $f(2.3)$ $= int(3 * 2.3)$
$= int(6.9)$
$= 6$

b.) $f(2)$ $= int(3 * 2) = int(6) = 6$

expl 2: For the function $f(x) = \dfrac{1}{x}$, find the following.

a.) $f(5)$ $= \dfrac{1}{5}$

b.) $f(0)$ $= \dfrac{1}{0}$ no real #
undefined
does not exist

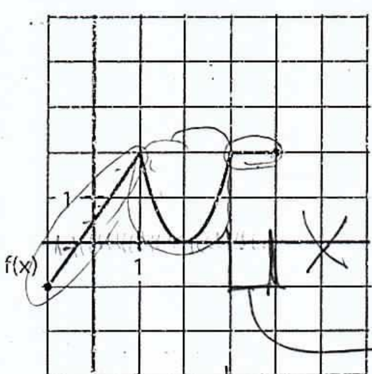
expl 3: For the function $f(x) = 5$, find the following.

a.) $f(2.3)$ $= 5$

b.) $f(2)$ $= 5$

Piecewise Functions:

The following is an example of a piecewise function. The idea here is that the function's *rule changes depending on which piece of the domain you're in.*



$3 \le x \le 4$

$-1 \le x \le 1$ $1 < x < 3$

First, verify that this is, indeed, a function. ✓

What is the domain of this function?

$[-1, 4]$ OR $-1 \le x \le 4$

Break up this graph into its three pieces and determine the x-values (domains) for those pieces.

The rule for this function has to come in three pieces, just as its graph does. Its formula is

$$f(x) = \begin{cases} \dfrac{3}{2}x + \dfrac{1}{2}, & -1 \le x \le 1 \\ 2(x-2)^2, & 1 < x < 3 \\ 2, & 3 \le x \le 4 \end{cases}$$

Notice how the domains for each piece do *not* overlap.

expl 4: For the function below to the right, complete the following.

a.) Find the domain.
b.) Locate the intercepts.
c.) Graph the function.
d.) Find the range based on the graph.

$$g(x)=\begin{cases}3x+3, & x<0\\ x+5, & x\geq 0\end{cases}$$

You graph this piecewise function by graphing your first rule for $x<0$ and your second rule for $x\geq 0$.

You can pick any points to graph but you should include endpoints and intercepts when you can.

(a) domain: $(-\infty, \infty)$

(b) y-intercept: $g(0)=0+5$ (using $g(x)=x+5$, $x\geq 0$)

$g(0)=5$ → $\boxed{\text{y int}: (0,5)}$

x-intercept: (Sub 0 in for y, solve for x)

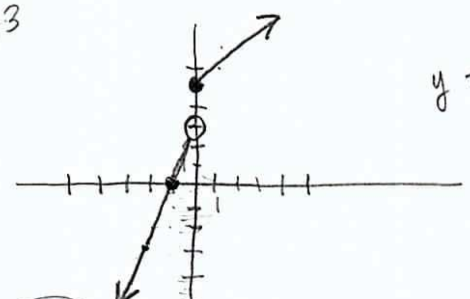
$y=3x+3$ OR $y=x+5$

$0=3x+3$ $0=x+5$

$-3=3x$ $\cancel{-5=x}$

$-1=x$ ✓   does not match domain given $(x\geq 0)$

$y=\frac{3}{1}x+3$



$y=\frac{1}{1}x+5$

(d) range

$(-\infty, 3)\cup[5, \infty)$

In MML, you will pick the graph from a multiple-choice list.

expl 5: For the piecewise function, find the function values $g(-10)$, $g(-15)$, and $g(20)$.

$$g(x)=\begin{cases}x+5, & x<-10\\ 3x-6, & -10\leq x\leq 0\\ 7, & x>0\end{cases}$$

Figure out where in the domain your x value is.

$g(x)=3x-6$

$g(-10)=3(-10)-6$

$g(-10)=-36$

↓

the point $(-10,-36)$

$g(-15)=-15+5$

$g(-15)=-10$

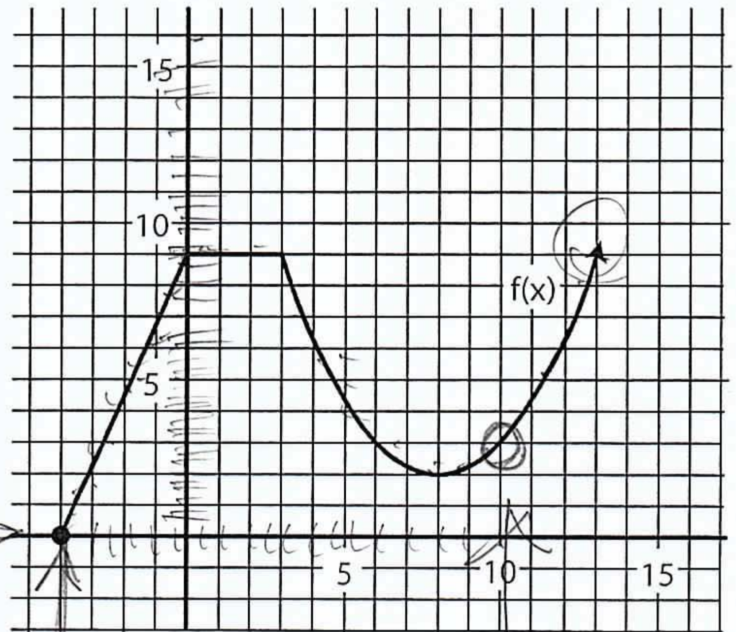
$g(20)=7$

3.4 x y

expl 6a: Determine the domain and range of the piecewise function pictured here.

dom: $[-4, \infty)$

range: $[0, \infty)$



expl 6b: Find $f(10)$.

$f(10) = 3$

$y=0$

$x=-4$

What y did they plot for $x=10$?

Worksheet: Piecewise Functions:

We will practice using and graphing piecewise functions.