Meet the straight line
and its equation.

College algebra
Linear Functions and Models (section 4.1)

Definition: Linear Equation: a linear equation in two variables is an equation that *could be written* in the form $Ax + By = C$ where A, B, and C are real numbers and A and B are *not* both zero.

$A=0$
$B=1, C=4$

Why would we say that A and B are *not* both zero?

Which of the following are linear equations?

$3x + 4y = 15$ ✓ $\quad 1y = 4$ ✓ $\quad 0x + 1y = 4$

$\cancel{4x^2 + 5y = 12}$ not linear $\quad y = 3x + 4 \rightarrow -3x + 1y = 4$ ✓

Different Forms: A linear equation could be written in many different forms. Each form has its own advantages. We will use the various forms to write equations depending on what information we are given and our preferences.

fairly easy to find intercepts

	General Equation	Example
Standard Form	$Ax + By = C$	$3x + 4y = 12$
Slope-Intercept Form	$y = mx + b$ or $f(x) = mx + b$	$y = \dfrac{-3}{4}x + 3$
Point-Slope Form	$y - y_1 = m(x - x_1)$	$y + 3 = \dfrac{-3}{4}(x - 8)$

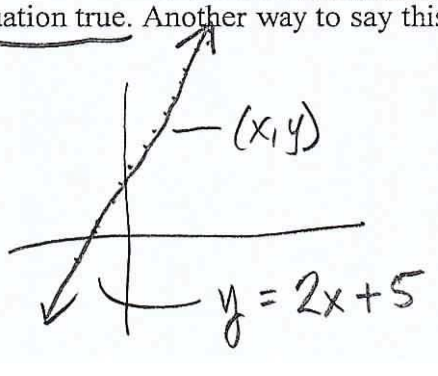
slope and y-intercept easy to pick out

slope and one particular point (relatively) easy to pick out

All of these equations describe the same line!

It also helps to think of (x, y) as a generic point on the line.

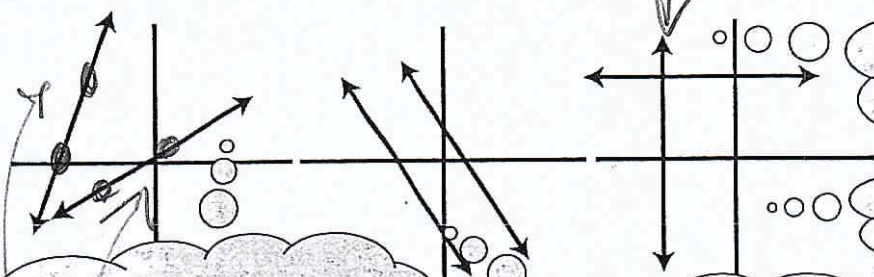
We will investigate graphs of linear equations here. Particularly, why are they straight and what makes them slanted? Remember, the idea behind a graph is that it shows every single point that makes the equation true. Another way to say this is that the points "satisfy the equation".



Points are in the form (x, y).

vertical line is not a fnc.

Slope: The slope of a line tells you how slanted it is. Imagine walking up (or down) a line from left to right and you understand why that is important.



Lines can be vertical or horizontal. Some lines are perpendicular.

All are functions except one. Which?

$x \in \mathbb{R}$

Lines can go up from left to right. Some are steeper than others.

Lines can go down from left to right. Some lines are parallel.

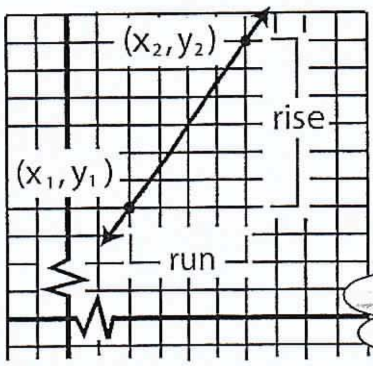
What is the domain of any non-vertical line?

all real #s. OR $(-\infty, \infty)$

Imagine any two points on a line. **Slope is the ratio of how far we go up (or down) to how far we go right (or left) to get from one point to the other point.** As the steepness of the line changes, this ratio would change too.

Formula for Slope:

The slope between the two points (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$.



Subscripts denote first and second points.

Also denoted as $m = \dfrac{\Delta y}{\Delta x}$.

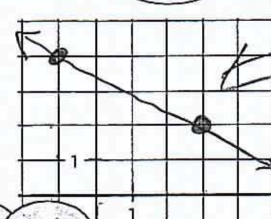
$slope = {}^{rise}/_{run}$

$rise$ = difference of y values

run = difference of x values

It does *not* matter which point you call (x_1, y_1).

expl 1: Find the slope of the line that goes through the points $(-1, 4)$ and $(3, 2)$. Plot the points on the graph and count the rise and run from one point to the other.



line goes down from left to right. (decreasing)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$= \frac{4-2}{-1-3} = \frac{2}{-4}$$

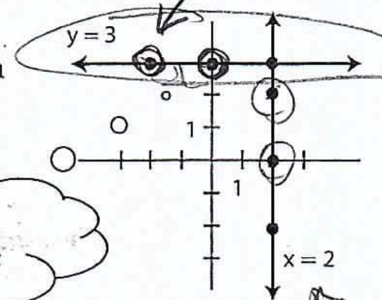
$$m = \frac{-1}{2}$$

Should the slope be negative or positive?

OR $\dfrac{1}{-2}$ or $-\dfrac{1}{2}$

$m = \frac{rise}{run} = \frac{0}{2} = 0$

Horizontal and Vertical Lines:
Find the slopes of these lines. Rather than using the formula from the previous page, use the quicker "rise over run" method.



$m = rise/run$

$slope = rise/run$

$m = \frac{rise}{run} = \frac{2}{0}$
undefined

Use what you found above to generalize about the slope of all vertical and horizontal lines.

Slope of any vertical line = undefined

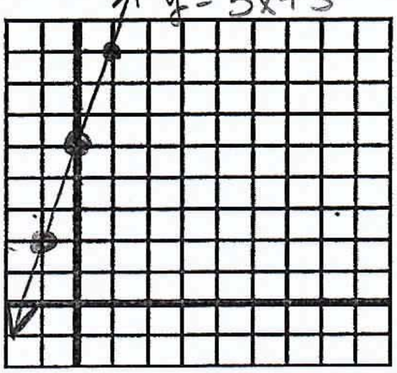
Slope of any horizontal line = 0

Recall: Slope-intercept Form of a Line:
Any (non-vertical) line could be written in the form $y = mx + b$. Here, m is the slope and b is the y-intercept. It also helps to think of (x, y) as a generic point on the line.

expl 2a: Find the slope and y-intercept of the line $h(x) = 3x + 5$. Graph the line below. (Assume a scale of one unit per tick mark.)

Get the function from MML and try out the grapher.

$y = 3x+5$



$y = 3x + \underline{5}$

$y = mx + \underline{b}$

$m = 3 = \frac{3}{1}$

expl 2b: Is this function increasing, decreasing, or constant? What about the equation tells you?

The average rate of change of a linear fnc is its slope.

It has a positive slope ($m = 3$).

Average Rate of Change and Graphs:

We have previously seen the formula for average rate of change (of f from a to b) to be

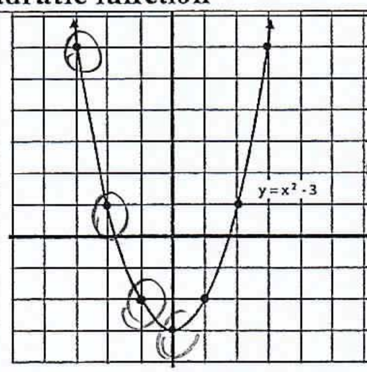
$$\frac{\Delta y}{\Delta x} = \frac{f(b)-f(a)}{b-a}, \quad a \neq b.$$

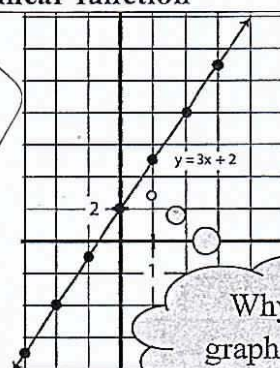
If the function in question is a linear function, then this average rate of change is nothing more than the slope of the line.

Therefore, when we are analyzing a generic function, if this average rate of change is found to be constant for whatever points we choose, the function must be linear. On the other hand, if we find the average rate of change for a function using several pairs of points and it is *not* constant, then we will say the function must *not* be linear.

expl 3: Consider the functions and their tables of values below.

Quadratic function		Linear function	
x	$y = x^2 - 3$	x	$y = 3x + 2$
-3	6	-3	-7
-2	1	-2	-4
-1	-2	-1	-1
0	-3	0	2
1	-2	1	5
2	1	2	8
3	6	3	11





Why is this graph straight?

Use the space below to find the average rate of change for these two functions between each pair of points. I have curtailed the tables to make our job easier.

$\frac{f(b)-f(a)}{b-a}$

Quadratic	
x	$y = x^2 - 3$
-3	6
-2	1
-1	-2
0	-3

Average Rates of Change

$\frac{1-6}{-2+{}^{+}3} = \frac{-5}{1} = -5$

$\frac{-2-1}{-1+{}^{+}2} = \frac{-3}{1} = -3$

$\frac{-3+{}^{+}2}{0+{}^{+}1} = \frac{-1}{1} = -1$

Linear	
x	$y = 3x + 2$
-3	-7
-2	-4
-1	-1
0	2

Average Rates of Change

$\frac{-4+{}^{+}7}{-2+{}^{+}3} = \frac{3}{1} = 3$

$\frac{-1+{}^{+}4}{-1+{}^{+}2} = \frac{3}{1} = 3$

$\frac{2+{}^{+}1}{0+{}^{+}1} = \frac{3}{1} = 3$

How can you identify a linear function by its average rate of change?

It will have a constant average rate of change (its slope, in fact). For $y = 3x+2$, the avg. rate of change is always 3.

~~Find f(0)~~

expl 4: Let $f(x) = \frac{1}{2}x + 1$ and $g(x) = \frac{-1}{2}x + 7$. Answer the following questions.

a.) Solve $f(x) = 0$. Where would this information be on the graph of $f(x)$?

$0 = \frac{1}{2}x + 1$

$-1 = \frac{1}{2}x$

$-2 = x$

The x-value when f(x) or y is 0.

We are finding the x-intercept of the fnc f(x).

b.) Solve $f(x) = g(x)$.

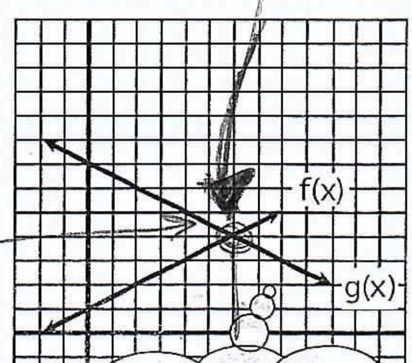
$\frac{1}{2}x + 1 = \frac{-1}{2}x + 7$

$+\frac{1}{2}x \quad -1 \qquad +\frac{1}{2}x \quad -1$

$1x = 6$

$x = 6$

c.) Here, we see f and g graphed. Find the point that represents the solution to $f(x) = g(x)$.



$x = 6$

We want the x-value where these functions' y-values are equal.

When you divide (or multiply) by a negative, you need to flip the sign.

Solving Linear Inequalities:

Solving linear inequalities is identical to solving linear equations, *except* when you do what? Do you remember? Let's see if it comes up in these examples.

expl 5: Solve the inequality. Then check your solution by substituting a value from the solution set into the original inequality. Does it work? If *not*, why *not*?

$-9x < 81$

$\frac{-9x}{-9} \overset{?}{<} \frac{81}{-9}$

$x > -9$

Check: (Choose $x = -8$)

$-9x < 81$

$-9(-8) \overset{?}{<} 81$

$72 \overset{?}{<} 81$ ✓

Do you remember the hitch when you divide or multiply an inequality by a negative? What needs to be done?

expl 6: Solve the inequality. Then check your solution by substituting a value from the solution set into the original inequality. Does it work? If *not*, why *not*?

$\frac{x}{3} \geq 12$

$\frac{\cancel{3}}{1} \cdot \frac{x}{\cancel{3}} \geq 12 \cdot 3$

$x \geq 36$

Check: (Choose $x = 36$)

$\frac{36}{3} \overset{?}{\geq} 12$

$12 \overset{?}{\geq} 12$ ✓

expl 7: Let's return to our functions $f(x) = \frac{1}{2}x + 1$ and $g(x) = \frac{-1}{2}x + 7$ from before. Can you solve the following inequalities? Write your answers in interval notation.

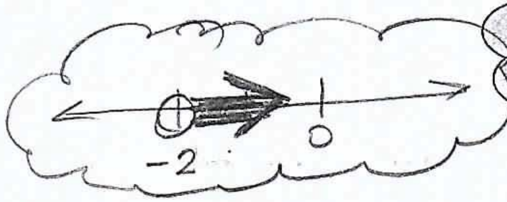
Plug the functions into the inequalities and solve.

a.) Solve $f(x) > 0$.

$\frac{1}{2}x + 1 > 0$

$\frac{1}{2}x > -1$

$x > -2 \rightarrow (-2, \infty)$

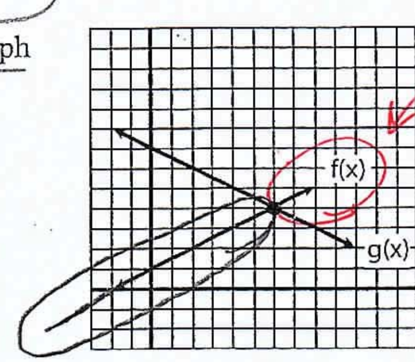


b.) Solve $f(x) \leq g(x)$. Circle this solution set on the graph to the right.

$\frac{1}{2}x + 1 \leq -\frac{1}{2}x + 7$

$-\frac{1}{2}x - 1 \quad +\frac{1}{2}x - 1$

$x \leq 6$



here $f(x) = g(x)$

So, $f(x) \leq g(x)$ for all x such that $x \leq 6$.

Definition: Zero versus x-intercept:

The **x-intercept** of a graph is usually written in ordered pair notation because it is thought of as a point. The **zero** of the relationship is the **x-value** of this point. Remember this is simply the x-value that makes the y-value equal to 0.

For a function given in $f(x)$ form, like $f(x) = 2x + 4$, how would you find its zero? Do it now.

$0 = 2x+4$

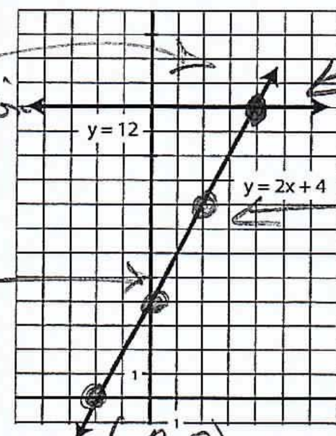
$-4 = 2x$

$-2 = x$

This idea will follow us throughout algebra.

Revisiting Solving Linear Equations and Inequalities Graphically:

expl 8: On the graph below, label the points $(-2, 0)$, $(0, 4)$, $(2, 8)$, and the point of intersection of the two lines. Label this intersection in ordered pair notation.



(4,12)

(2,8)

(0,4)

(-2,0)

a.) Algebraically, solve the equation $2x + 4 = 12$. Where in the graph here do you see your solution? Why?

$2x+4=12$

$2x=8$

$x=4$

It's the intersection of $y=2x+4$ and $y=12$.

b.) The line $y = 2x + 4$ is a function. (Why?) Let's rename it $f(x)$. Use the graph of $f(x) = 2x + 4$ to solve the following graphically.

i.) Solve $f(x) = 0$.

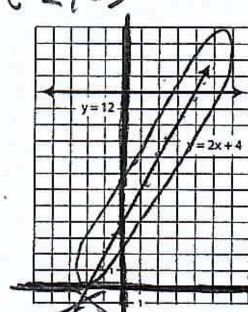
Find x-intercept $(x=-2)$

ii.) Solve $f(x) = 8$.

On graph, find point where $y=8$ $(x=2)$

iii.) Solve $f(x) > 0$. Use interval notation. Circle this solution on the mini-graph.

$(-2, \infty)$

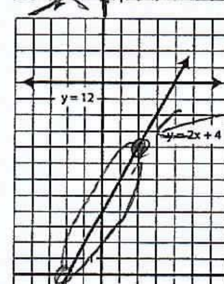


for part iii

iv.) Solve $0 < f(x) < 8$. Use interval notation. Circle this solution on the mini-graph.

$-2 < x < 2$

or $(-2, 2)$



(2,8)

for part iv

(-2,0)

4.1

$S(p)$ = quantity supplied of T-shirts
$D(p)$ = " demanded " "

expl 9: Suppose that the quantity supplied S and the quantity demanded D of T-shirts at a rally are given by the following functions. Let p represent the price of a shirt in dollars. Answer the following questions.

$S(p) = -500 + 40p$ p = price of shirt (\$)

$D(p) = 1100 - 30p$

a.) Find the equilibrium price of the shirt. What is the equilibrium quantity?

supply = demand

$-500 + 40p = 1100 - 30p$

$+30p \qquad +30p$

$+500 \qquad +500$

$70p = 1600$

$\frac{70p}{70} = \frac{1600}{70}$

$p \approx \$22.86$

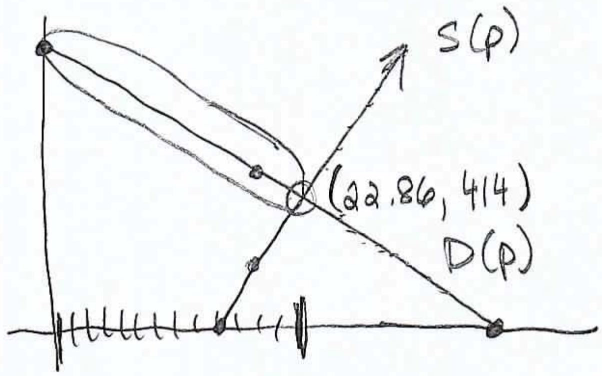
$S(p) = -500 + 40p$

$S(22.86) = -500 + 40(22.86)$

$S(22.86) \approx 414$ shirts

The equilibrium point is where demand equals supply. Which variable is price and which is quantity?

b.) Draw a quick graph of the two functions, labeling them. Consider their domains.



Any old line can go on forever. But these lines represent something in the real world. What values can p take on?

c.) Circle the part of the graph where Demand is greater than Supply. Write an inequality for these p-values.

$0 \le p < 22.86$ \$

If demand is greater than supply, the price will go up, won't it?

Worksheet: Roots and Intersections on your Calculator (82, 83, 84, 85, 86):
We will learn how to solve equations graphically using the calculator.