We will build equations from verbal descriptions or data.

College algebra
Class notes
Building Quadratic Models and Regression (section 4.4)

We will see how some story problems result in a quadratic equation. We can use what we know about the graphs of parabolas (x-intercepts, extrema, etc.) to answer questions. In this section, we also see how a calculator can be used to find the quadratic function that best fits a scatter plot.

Quadratic Models:

Perimeter

expl 1: Marison has 500 feet of track she will be using to make a rectangular remote-control car racetrack. She wants the area enclosed by the track to be the largest possible. What should the width and length be? Follow these steps.

a.) Solve the perimeter formula (of a rectangle) for the length, l. Finish this by letting P be 500 feet.

$$P = 2l + 2w \rightarrow P = 2l + 2w$$

$$-2w \quad -2w$$

$$P - 2w = 2l$$

$$\frac{P-2w}{2} = \frac{\cancel{2}l}{\cancel{2}}$$

$$l = \frac{P-2w}{2}$$

$$l = \frac{500-2w}{2}$$

$$l = 250 - w$$

w, l

P = perimeter
l = length
w = width

b.) Express the area A as a function of the width, w.

$$A = l \cdot w$$

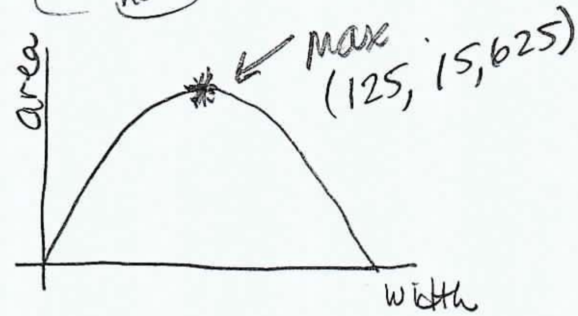
$$A = (250-w) \cdot w$$

$$A = 250w - w^2 \quad \text{OR} \quad \boxed{A(w) = 250w - w^2}$$

{ f(x) notation

c.) Graph this area function. Think about various possibilities for width and length, and their areas, to determine an appropriate window.

$[0, 250] \times [0, 20{,}000]$



d.) What should the width be to maximize the area inside the track? What is the maximum area? Include units.

The width is 125 feet.
The max. area is 15,625 ft^2.

expl 2: The price p, in dollars, and the quantity x sold of a certain product are related by the following equation. Answer the questions that follow.

$\boxed{x = -4p + 440}$

p = price (\$)
x = quantity sold

Revenue, the total money they make, is price times quantity sold.

a.) Find a model that represents the revenue R for this product as a function of price p.

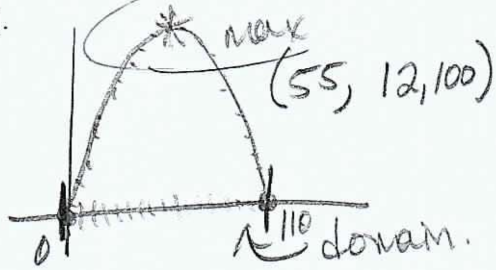
$R = x \cdot p$

$R = (-4p + 440) \cdot p$

$R(p) = -4p^2 + 440p$ or $R = -4p^2 + 440p$

p = price (\$)
R = revenue (\$)

b.) Graph this revenue function on the window [0, 150] x [0, 15,000].



c.) Revenue is the money they make, so assume it to be non-negative. What is the domain of this function $R(p)$?

$R = (-4p + 440) \cdot p$

$0 = (-4p + 440) \cdot p$

$0 = -4p + 440$ or $0 = p$

$p = 110$

domain: $[0, 110]$

d.) Let's investigate the maximum point of this parabola.

i.) What price maximizes the revenue?

$p = \$55$

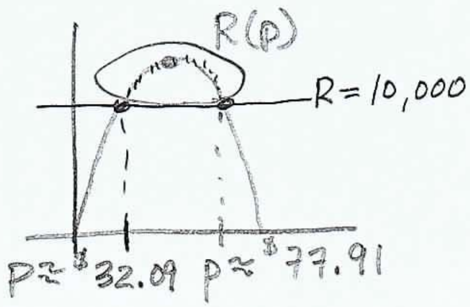
ii.) What is the maximum revenue?

$R(p)$ or $R = \$12,100$

iii.) How many units are sold at this price? — or $x = -4p + 440 = -4(55) + 440 = 220$

$\text{quantity} = \dfrac{R}{p} = \dfrac{12,100}{55} = 220$ units

e.) If the company needs at least \$10,000 revenue for this product, what price range should they charge? Label this on the graph as well.



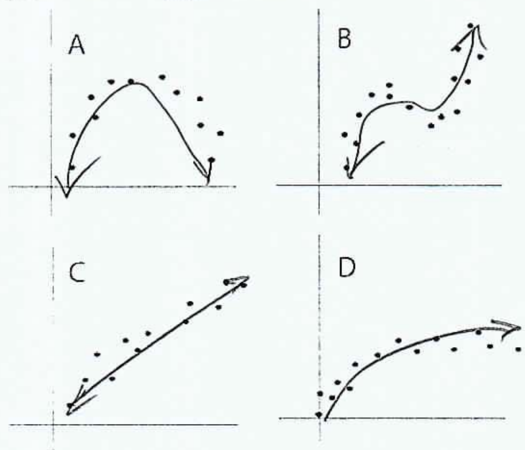
The price range should be (\$32.09, \$77.91).

(or $\$32.09 < p < \$77.91$)

Quadratic Regression Equations:

Later, we will see cubic ($y = x^3$ ...) regression.

We studied regression before. We saw how the pattern of a scatter plot of points could be represented by a single linear equation. But *not* all scatter plots show a linear pattern. Look at the plots below. Draw in a curve (or line) that mimics the pattern of points.



Which looks like a quadratic pattern (parabola)?

Graph A

expl 3a) The number of foreign adoptions in the U.S. has declined in recent years, as shown in the table to the right.

i.) Use your calculator to draw a scatter plot and then fit a quadratic function to this data. Let x represent the number of years since 2000. Round your equations' coefficients to three decimal places.

$y = ax^2 + bx + c$

$y = -342.258x^2 + 2687.052x + 17{,}133.109$

Year, x	**Number of U.S. Foreign Adoptions from Top 15 Countries, y**
2000, 0	18,120
2001, 1	19,087
2002, 2	20,100
2003, 3	21,320
2004, 4	22,911
2005, 5	22,710
2006, 6	20,705
2007, 7	19,741
2008, 8	17,229
2009, 9	12,782

Take note of how they define x.

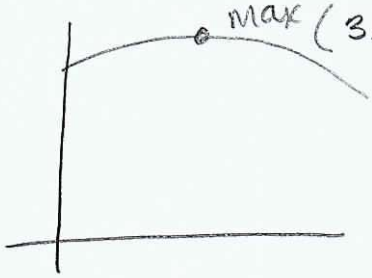
$x = 10$

ii.) Use the function from part a to estimate the number of U.S. foreign adoptions in 2010.

$y = -342.258(10)^2 + 2687.052(10) + 17133.109$

$y \approx 9{,}778$ adoptions predicted in 2010 expl 3b: Use the calculator to find the maximum for the quadratic regression equation. Write a sentence or two to give meaning to the x and y values of this point.

max (3.9, 22,407)



The maximum number of adoptions during this period is 22,407 adoptions seen about 3.9 years after 2000.

expl 3c: Use your regression equation to estimate the number of adoptions in the year 2050. Why does this value *not* make sense? (Using your regression equation to predict values well outside your original data set is called **extrapolation** and should *not* be done.)

$x = 50, \; y = -704{,}159.3$

So, in 2050, there will be −704,159 adoptions. That makes no sense.

This is *not* covered in our book but you might see it elsewhere.

Definition: Coefficient of Determination, R^2:

The **coefficient of determination** is used similarly to the correlation coefficient seen with linear regression. When we try to fit various types of regression (quadratic, cubic, or quartic) to a set of data, the coefficient of determination will tell us which one gives us the *best* fit. The regression equation that gives us an R^2 value closest to 1 will be the one we choose to use.

Worksheet: Quadratic (and higher order) Regression on Your Calculator (TI-82, 83, or 84):

This worksheet provides an example and step-by-step instructions for finding a quadratic, cubic, and quartic regression equations on your calculator. The higher degree regression equations are done similarly. On these problems, you will want to compare higher orders of regression to find which fits the data best. This is mentioned on the worksheet.

Cubic regression will be covered later. This book warns against using quartic regression.