We will see these rational functions in real-life scenarios.

College algebra
Class notes
Applications of Rational Functions (section 5.6)

We have learned about the vertical and horizontal asymptotes of rational functions. What meaning can we give them in real-life scenarios?

expl 1: The population P, in thousands, of a senior community is given by $P(t) = \dfrac{500t}{2t^2+9}$ where t is the time in months.

$P(t)$ = population in 1000's
t = time in months

a.) Find the horizontal asymptote of the graph and complete the statement
$P(t) \to \underline{0}$ as $t \to \infty$.

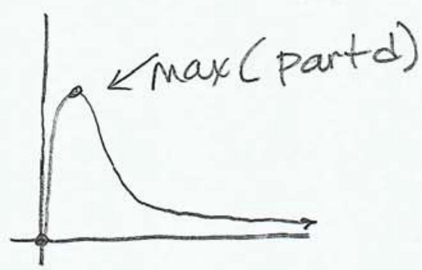
degree on top = 1, degree on bottom = 2
So HA. $y = 0$ (from section 5.5)

b.) Explain the meaning of the answer to part *a*.

As time goes on, the population of the community approaches 0. (Residents are either moving or dying.)

c.) Graph the function on the window [0, 25] x [0, 100]. Be sure to put the entire bottom in parentheses.

$y_1 = 500x/(2x^2+9)$



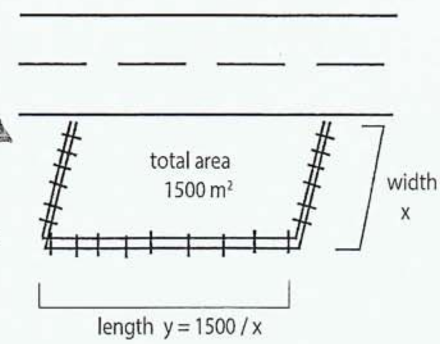
d.) Determine the time, to the nearest month, when the population is at its maximum.

maximum : (2, 58.926) (rounded)

(x, h(x))

The pop reached a maximum of 58.926 thousand (or 58,926 people) at around the 2 month mark. (Then it fell and fell and fell...)

expl 2: This rectangular corral alongside a highway must be fenced to have an area of 1500 square meters. We will *not* lay fencing along the highway side. Let x and y be defined as in the picture. There will be four corner posts which cost \$60 each. The fencing along the long side (labeled length) will cost \$25 per linear meter. The fencing along the two widths will cost \$15 per linear meter.



a.) Find the cost of this corral as a function of x, the width of the corral.

- four corner posts $= 4 * 60 = \$240$
- 2 widths cost $= 2 \cdot x \cdot 15 = 30x$ (dollars)
- 1 length cost $= y \cdot 25 = \frac{1500}{x} \cdot 25 = 37500/x$ (dollars)

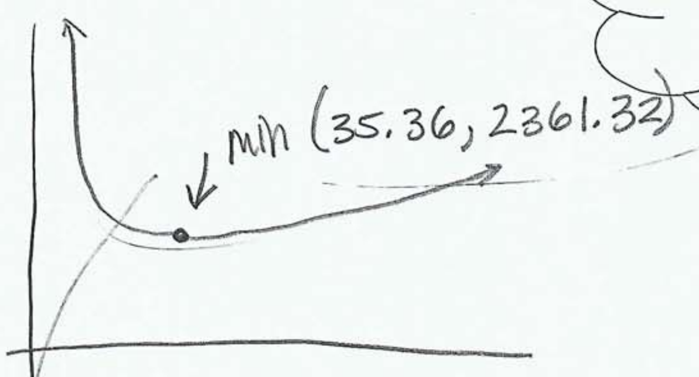
Add

$$\boxed{\text{Cost} = 240 + 30x + 37500/x}$$

b.) Graph your function. Use a large enough window so that you can see a minimum. Then find this minimum and interpret it.

$C(x) = 240 + 30x + 37500/x$

If you cannot find a good window, use the Value function under CALC to see what y is for a reasonable value of x like 50. Then change your Ymax accordingly.



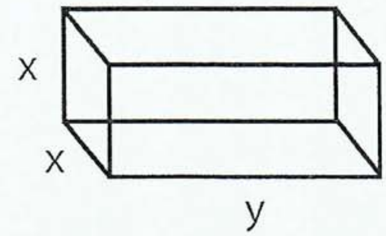
$[0,75] \times [0,5000]$

I started at $[0,50] \times [0,2500]$ and saw a bit of a graph at the top. Then I tried $[0,50] \times [0,5000]$. I then increased x-max to 75 to make the minimum clearer.

The minimum cost of \$2361.32 will occor when x (the width) is set at 35.36 feet.

has a top (6 faces total)

expl 3: Pictured to the right is a closed box we have been asked to make. We need a total volume of 12,000 cubic inches. Notice the end faces are squares (x by x inches) with the length along the third dimension labeled as y.



a.) Express the surface area (all six faces) as a function of x. Follow these steps.

i.) Define a volume (V) formula using x and y. Set V equal to 12,000 and solve for y.

$$V = x \cdot x \cdot y$$

$$12{,}000 = x^2 y$$

$$y = 12{,}000/x^2$$

x = width
x = height
y = length of box
) inches
V = volume (in^3)

ii.) Define a surface area (SA) formula using x and y. Substitute your expression for y and end up with a surface area formula in just x.

$$SA = \text{end faces' areas} + \text{long faces' areas}$$

There are 2 ends and 4 long faces.

$$= 2x^2 + 4xy$$

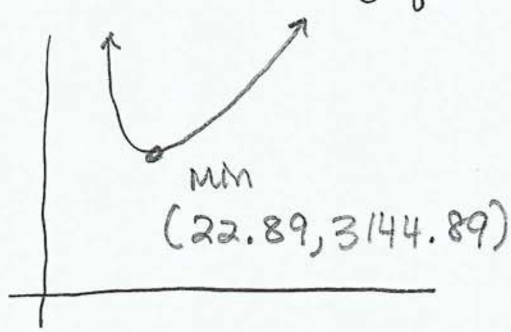
$$SA = 2x^2 + 4x\left(\frac{12000}{x^2}\right) = 2x^2 + 48{,}000/x$$

(sq. in.)

b.) Graph this surface area function and find its minimum.

i.) What is the least amount of cardboard that can be used to make this box? (SA)

We would need a minimum of 3144.89 square inches of cardboard.



[0,75] x [0,5000]

ii.) What are the dimensions of the box with the least surface area?

22.89 in x 22.89 in x 22.90 in

$y = 12000/22.89^2$

$y \approx 22.90$