These rules will help us manipulate logs.

College algebra
Class notes
Properties of Logarithms (section 6.5)

Optional Worksheet: Logarithm Rules Worksheet

This worksheet will show you how to derive most of the formulas given in this section. If we can think through the formulas, they will be easier to memorize and apply. Try to use what you know about logs to figure out the following excerpt from the worksheet.

$b^? = b$
$5^? = 5$

$b^? = 1$
$5^? = 1$
$5^0 = 1$

$b^? = b^k$

1. In words, what is $\log_b b$? It's the number to which I raise __b__ to get __b__.
What does this number we call $\log_b b$ have to be? $\log_b b = 1$

2. In words, what is $\log_b 1$? It's the number to which I raise __b__ to get __1__.
What does this number we call $\log_b 1$ have to be? $\log_b 1 = 0$

3. In words, what is $\log_b b^k$? It's the number to which I raise __b__ to get __b^k__.
What does this number we call $\log_b b^k$ have to be? $\log_b b^k = k$

4. Now $\log_b v$ is the number to which I raise b to get v. So, when we raise b to this number, what should I get? In other words, what is $b^{\log_b v}$? $b^{\log_b v} = v$

The complete list of rules is listed below including the Change of Base Formula from the end of this section.

1. $\log_a \left(M/N\right) = \log_a M - \log_a N$
2. $\log_a (M \cdot N) = \log_a M + \log_a N$
3. $\log_a M^r = r \cdot \log_a M$
4. $\log_a a = 1$
5. $\log_a 1 = 0$
6. $\log_a a^r = r$
7. $a^{\log_a M} = M$
8. $a^r = e^{r \ln a}$
9. $\log_a M = \dfrac{\log_b M}{\log_b a}$

(Change of Base Formula

The worksheet will give you ways to think through most of these rules to make sense of them.

exponent rule:

$\frac{a^m}{a^n} = a^{m-n}$

$a^m \cdot a^n = a^{m+n}$

Common Mistakes:

It is common to incorrectly assume other rules similar to those given. Be careful when you apply the rules. You should also try out numbers in any rule you "think" is right.

$\dfrac{\log_a M}{\log_a N} \neq \log_a M - \log_a N$

$(\log_a M)*(\log_a N) \neq \log_a M + \log_a N$

$\log_a MN \neq (\log_a M)(\log_a N)$

$(\log_a M)^p \neq p(\log_a M)$

etc...

Substitute values to check any rule.

$\dfrac{\log_{10} 100}{\log_{10} 1000} \stackrel{?}{=} \log_{10} 100 - \log_{10} 1000$

For all examples, assume that variables are such that logs are defined. For instance, we *cannot* take the log of a negative number.

expl 1: Express as a sum of logs. Simplify if possible.

$\log_4(64y)$

$\log_a(M \cdot N) = \log_a M + \log_a N$ (Rule 2)

$\log_4(64y) = \log_4 64 + \log_4 y$

$= 3 + \log_4 y$

expl 2: Express as a sum or difference of logs. Express powers as factors. Simplify if possible.

$\ln y^5$

Rule 3: $\log_a M^r = \underline{\underline{r}} \cdot \log_a M$

$= \log_e y^5$

$= 5 \cdot \log_e y$ OR $5 \cdot \ln y$

expl 3: Use the properties of logarithms to find the exact value of this expression.

$\log_8 2 + \log_8 4 = \log_8(2 \cdot 4)$ Rule 2

$= \log_8 8$

$= 1$

because $8^1 = 8$

or Rule 4

expl 4: Simplify.

a.) $t^{\log_t 3} = 3$

Rule 7

b.) $\ln e^6 = \log_e e^6 = 6$

this is the # I raise e to, to get e^6

Or Rule 6.

expl 5: Express as a sum or difference of logs. Express powers as factors. Simplify if possible.

a.) $\log\frac{x^2y}{(x+1)} = \log(x^2y) - \log(x+1)$ Rule 1

$= \log x^2 + \log y - \log(x+1)$ Rule 2

$= 2\cdot\log x + \log y - \log(x+1)$ Rule 3

Use $\log_a M^p = p\cdot\log_a M$ where applicable.

b.) $\log_5\frac{(x+2)^3}{x^4} = \log_5 (x+2)^3 - \log_5 x^4$ Rule 1

$= 3\cdot\log_5(x+2) - 4\cdot\log_5 x$ Rule 3

$\log_a M - \log_a N = \log_a (M/N)$

expl 6: Express as a single log. Express powers as factors. Simplify if possible.

What cancels?

a.) $\underbrace{\ln\left(\frac{x}{x-2}\right)-\ln\left(x^2-4\right)}_{\text{Rule 1}}+\ln\left(\frac{x+2}{x}\right)$

$= \underbrace{\ln\left(\frac{x}{x-2}\cdot\frac{1}{(x^2-4)}\right) + \ln\left(\frac{x+2}{x}\right)}_{\text{Rule 2}}$

$= \ln\left(\frac{x \cdot 1}{(x-2)(x^2-4)} \frac{(x+2)}{x}\right)$

$= \ln\left(\frac{\cancel{x}}{(x-2)} \frac{1}{(x-2)\cancel{(x+2)}} \cdot \frac{\cancel{(x+2)}}{\cancel{x}}\right)$

Rule 3

$= \ln\left(\frac{1}{(x-2)^2}\right) = \ln (x-2)^{-2} = \boxed{-2 \cdot \ln(x-2)}$

Do you remember the difference of two squares?

$a^2-b^2=(a-b)(a+b)$

$x^2-4=(x-2)(x+2)$

There are different ways this could be simplified.

b.) $\underbrace{3 \cdot \ln(x-5)}_{\text{Rule 3}}-\underbrace{\left[\ln(x-5)+\ln(x+5)\right]}_{\text{Rule 2}}$

Rule 1

$= \ln (x-5)^3 - \ln (x-5)(x+5)$

$= \ln\left(\frac{(x-5)^3}{(x-5)(x+5)}\right)$

$= \ln\left(\frac{(x-5)^2}{(x+5)}\right)$

Change of Base Formula:

You *cannot* find logs other than base 10 or e on your calculator. We need another way. For any logarithmic bases a and b, and any positive number M, we know that $\boxed{\log_b M = \dfrac{\log_a M}{\log_a b}}$.

> Some newer calculators will calculate logs of any base. Look under the **MATH** menu for **"logBASE"**.

[We will usually use 10 or e for the base a so we can do these problems on the calculator.]

expl 7: Use the Change of Base Formula and your calculator to find the following.
$\log_3 12$

$$\log_3 12 = \frac{\log_{10} 12}{\log_{10} 3}$$

$$\approx 2.26$$

> Do you know the number we raise 3 to, to get 12? If not, use the formula!

> You can use log base 10 or e. Try both to see!

(because $3^{2.26} = 12$)

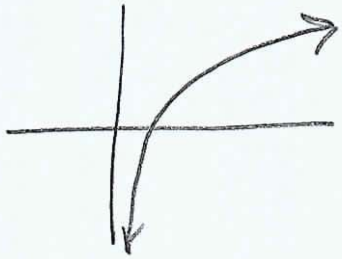
expl 8: Use the Change of Base Formula to rewrite the following and then evaluate *without* a calculator.

$$\log_3 5 \cdot \log_5 81 = \frac{\cancel{\log_3 5}}{\log_3 3} \cdot \frac{\log_3 81}{\cancel{\log_3 5}} = \frac{\log_3 81}{\log_3 3} = \frac{4}{1} = 4$$

expl 9: Graph on a calculator using the Change of Base Formula. Use a window of [-2, 5] x [-3, 3].
$y = \log_4 x$

$$y_1 = \log_{10}(X) / \log_{10}(4)$$

> This question will be multiple choice.



$y = \log_4 x$

$x = 16$

Solving Special Logarithmic Equations:

What would you say the solution to $\log_2 x = \log_2 16$ is? Go with your gut. There is actually a pretty simple property that backs up what your gut probably told you.

Logarithmic Equality Property: For any $M > 0,\ N > 0,\ a > 0$, and $a \neq 1$, we know that $M = N$ if and only if $\log_a M = \log_a N$.

Do you remember what "if and only if" means?

expl 10: Express y as a function of x. The constant C is a positive number.

$\log y = 2 \cdot \log x - \log C$ Rule 3

$= \log_{10} x^2 - \log_{10} C$ Rule 1

$\cancel{\log_{10} y} = \cancel{\log_{10}} \left(\frac{x^2}{C}\right)$ Log Equal Prop.

$$y = \frac{x^2}{C}$$

Recall a missing base implies "10".

expl 11: It is easy to remember these rules incorrectly. We may need to check our memory. Substitute values for the variables to verify that the following "rule" is true. Use a base of 10 or e so you can use your calculator to evaluate the logs. or not

$\frac{\log_a M}{\log_a N} \overset{?}{=} \log_a M - \log_a N$

$$\underbrace{\frac{\log_{10} 20}{\log_{10} 3}}_{\text{calc}} \overset{?}{\neq} \underbrace{\log_{10} 20 - \log_{10} 3}_{\text{calc.}}$$

2.73 0.82