Here are some real-life applications of the exponential function.

College algebra
Class notes
Financial Models (section 6.7)

We are going to invest an amount of money (**principal**, **present value**) into a bank account that pays **compound interest**. We will find the amount of money at the end of some time period (**accumulated value**). Sometimes, we frame this as someone **borrows** P dollars and is charged compound interest.

You may have studied **simple interest** in the past.

Consider a principal of $1,000 invested at 5% *simple interest* for 4 years. The money earns interest at the rate of 5% of the initial deposit every year for 4 years. Hence, $I = PRT = 1000 \times .05 \times 4 = \$200$. After four years, they have a total of $1,200.

With simple interest, interest is based on how much was initially deposited. But after one year, we have an extra $50 (the interest earned or 5% of $1,000) that we could invest. *If* we put that money back into the account and let it earn interest, we call that **compound interest**. All interest that is earned is subsequently reinvested to start earning the same rate as the initial deposit. The formula for that is here.

Compound Interest Formula:

$$A = P\left(1+\frac{r}{n}\right)^{n \cdot t}$$

P = initial deposit (principal)
A = amount in account after t years
r = annual interest rate (decimal form)
n = number of compoundings per year
t = number of years

This assumes the account is compounded in one of several common ways. We could compound an account yearly (n = 1 time a year), semiannually (n = 2 times a year), quarterly (n = 4), monthly (n =12), weekly (n = 52), or daily (n = 365).

And then, there's compounding continuously. Here, we imagine that n gets larger and larger; that is, $n \to \infty$. The formula actually becomes simpler.

Continuously Compounded Interest Formula:

$$A = Pe^{r \cdot t}$$

The variables mean the same but notice we are using the irrational number *e*.

$n = 4$ decimal $= 0.034$

expl 1: Use the compound interest formula to answer the following questions.
a.) Suppose \$150,000 is deposited in an account that pays 3.4% compound interest, compounded quarterly. Find the function for the amount $A(t)$ in the account after t years.

$$A = P(1 + r/n)^{nt}$$

$$A(t) = 150{,}000\left(1 + 0.034/4\right)^{4t}$$

using fnc notation like f(x)

$$A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

P = initial deposit
A = amount in account after t years
r = interest rate (decimal form)
n = number of compoundings per year
t = number of years

b.) How much is in the account after 4.5 years?

$$A(4.5) = 150{,}000\left(1 + 0.034/4\right)^{(4(4.5))}$$

parentheses on calculator

$$\approx \$174{,}685.76$$

Solving for Present Value:
expl 2a: Solve the continuously compounded interest formula for P.

$$A = Pe^{r \cdot t}$$

This will be used when we know how much we want in so many years but do *not* know how much to invest now.

$$\frac{A}{e^{rt}} = \frac{Pe^{rt}}{e^{rt}}$$

$$\boxed{P = \frac{A}{e^{rt}} \quad or \quad P = A \cdot e^{-rt}}$$

For non-continuously compounded accounts:

$$P = A\left(1 + \frac{r}{n}\right)^{-n \cdot t}$$

expl 2b: Margot needs to accumulate \$50,000 in ten years. How much should she deposit now in an account that pays 2.3% interest compounded continuously? P=?

$$P = A e^{-rt}$$

$$= 50{,}000\, e^{-0.023(10)}$$

$$\boxed{P \approx \$39{,}726.68}$$

decimal
$1.25\% = 0.0125$

Solving for Present Value Again:

We will use the formula $P = A\left(1+\frac{r}{n}\right)^{-n\cdot t}$ to find the present value (principal) needed to invest now in order to accumulate a certain amount (A) in the future.

expl 3: Find the principal needed now to accumulate \$3,000 (A) on 3.5 years (t) if the account earns 1.25% (r) compounded daily. → $n = 365$

$$P = A(1 + r/n)^{-nt}$$

$$= 3000(1 + 0.0125/365)^{(-365 * 3.5)}$$

$$P \approx \$2871.58$$

Solving for Time:

We will see that the variable we are solving for is in the exponent position. This will require our knowledge of solving exponential equations acquired earlier.

expl 4: Marcos wants \$2500 (A) to go on vacation. He has \$1500 (P) he will invest in an account that will compound continuously at 5% to earn his vacation fund. How many years (to the nearest tenth) will it take? → $A = Pe^{rt}$ $\quad r = 0.05 \quad t = ?$

$$A = Pe^{rt}$$

$$2500 = 1500e^{0.05t}$$

$$\frac{2500}{1500} = \frac{\cancel{1500}e^{0.05t}}{\cancel{1500}}$$

$$1.67 \approx e^{0.05t}$$

$$\ln(1.67) \approx \ln e^{0.05t}$$

$$\ln(1.67) \approx 0.05t \cdot \cancel{\ln e}\ !$$

$$\frac{\ln(1.67)}{0.05} \approx \frac{\cancel{0.05}t}{\cancel{0.05}}$$

$t \approx 10.2$ years

★ Do not do $\ln(1.67)$ but rather calculate $2500/1500$ and then do $\ln(\text{Ans})$ on calculator so you only round at the end. ★

Inflation:

Inflation means the erosion of purchasing power of money. For example, if the annual inflation rate is 3%, then \$100 worth of purchasing power now will have only \$97 worth of purchasing power in a year.

If the rate of inflation averages r % per year over n years, the amount A that P dollars will purchase after n years is $A = P(1-r)^n$.

expl 5: If the average inflation rate is 2%, how long will it be until purchasing power is cut in half?

$A = P(1-r)^n$

$50 = 100(1-0.02)^n$

$0.5 = 0.98^n$

$\ln 0.5 = \ln 0.98^n$

$\ln 0.5 = n \cdot \ln 0.98$

$n = \ln 0.5 / \ln 0.98$

$\boxed{n \approx 34.3 \text{ years}}$

Put 2% in for r. You can use a specific value for P like \$100. What would A be if this purchasing power were cut in half?

We solve for n. How do we do that?

take ln (natural log) of both sides!

Effective Rate of Interest:

The **effective rate of interest (r_E)** is the annual simple interest rate that would yield the same amount as compounding n times per year, or continuously, after 1 year. Our formulas are below.

$$r_E = \left(1 + \frac{r}{n}\right)^n - 1$$

$$r_E = e^r - 1$$

The **first formula** is for if the account is compounded any other way but continuously. The **second formula** is for accounts that are compounded continuously.

$r_E = ?$

expl 6: Find the effective rate of interest for the following account. Round to the nearest thousandths of a percent.

5% compounded quarterly $r = 0.05$ $n = 4$

$r_E = (1 + r/n)^n - 1$

$= (1 + 0.05/4)^4 - 1$

$r_E \approx 0.05095$

5.095 %