

College algebra

Class notes

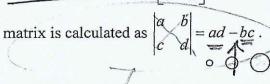
Matrices: Determinants (section 8.3)

Determinants can be used to solve systems of equations by using something called Cramer's Rule (sometimes, but not always). There are other uses like for certain integration problems in calculus and certain geometric calculations but we won't go there.

The process of finding a determinant for a matrix gives us a single value. (It is actually a function since you get only one determinant for a given matrix.) We will play around with this idea to get our toes wet but not go much further. There is a fairly simple calculation, at least for 2x2 matrices. After that, it gets ugly... quite ugly... and we will use the calculator.

## Formula for the Determinant of a 2x2 Matrix:

Define a square matrix  $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ . Here, a, b, c, and d are real numbers. The determinant of this



Notice the difference between a matrix's brackets and its determinant's brackets.

expl 1: Find the determinant below.

$$\begin{vmatrix} 3 \\ 1 \end{vmatrix} = 15 - 2 = 17$$

expl 2: Find the determinant below.

$$\begin{vmatrix} 7 & 14 \\ 2 & 4 \end{vmatrix} = 28 - 28 = 0$$

## Finding Determinants on the Calculator:

You must enter a square matrix (same number of rows and columns). Otherwise, it will give you an error.



- 1. Enter the MATRIX menu. It is the 2<sup>nd</sup> function of the x<sup>-1</sup> button. Arrow over to EDIT.
- 2. Select [A] and press ENTER. You will then enter the order of the matrix, rows x columns.
- 3. Fill in the matrix with its entries.
- 4. Quit out to the home screen and re-enter the MATRIX menu. This time arrow over to MATH. Select 1:det(. This is at the top of the list.
- 5. Re-enter the MATRIX menu and select [A] from the NAMES list.
- 6. Your home screen should now read **det([A]**. Press **ENTER** and it will output the matrix's determinant.

## Formula for Determinant of 3x3 Matrix:

Nah, I'm joking. We aren't doing that by hand. Do those with the calculator.



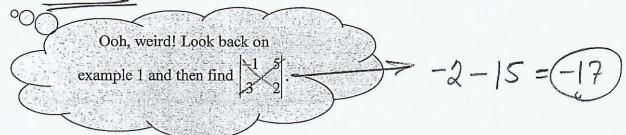
expl 3: Find the determinant below.



## |32 | = 17 ( from pg )

Properties of Determinants (or, What Some Smart People Learned Long Ago About Determinants):

1. If any two rows (or any two columns) of a matrix are interchanged, then the value of the determinant changes sign.



(2.) If all the entries in any row (or any column) equal 0, the value of the determinant is 0.

Aha! Make up such a 2x2 matrix and find its determinant.

$$\frac{|4-3|}{|60|} = 0 - 0 = 0$$
OR  $\frac{|50|}{|70|} = 0 - 0 = 0$ 

3. If any two rows (or any two columns) of a determinant have corresponding entries that are equal, the value of the determinant is 0.

Let's prove this one! Find the

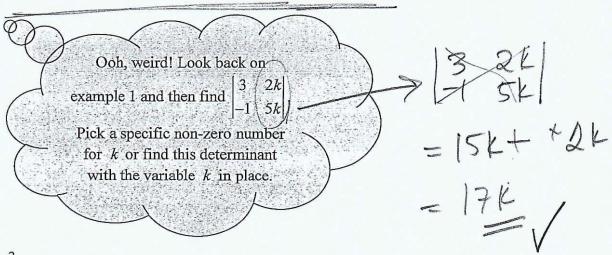
determinants 
$$\begin{vmatrix} a & b \\ a & b \end{vmatrix}$$
 and  $\begin{vmatrix} a & a \\ b & b \end{vmatrix}$ .

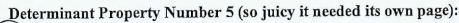
$$|ab| = ab - ab = 0$$

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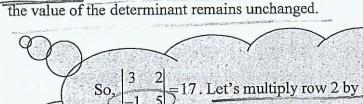
$$|ab| = ab - ab = 0$$

4. If any row (or any column) of a determinant is multiplied by a non-zero number k, then the value of the determinant is also changed by a factor of k.



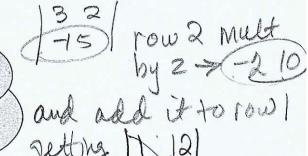


5. If the entries of any row (or any column) of a determinant are multiplied by a non-zero number k and the result is added to the corresponding entries of another row (or column), then



2 and add it to row 1, getting  $\begin{vmatrix} 1 & 12 \\ -1 & 5 \end{vmatrix}$ 

What is this new determinant?



= 5 -- 12

expl 4. It is known that  $\begin{vmatrix} x & y & z \\ u & v & w \\ 1 & 2 & 3 \end{vmatrix} = 4$ . Find the values of the following determinants using their

properties discussed above. Give a brief explanation; which property(s) are you using?

(a.) 
$$\begin{vmatrix} u & v & w \\ x & y & z \\ 1 & 2 & 3 \end{vmatrix} = -4$$

meter changed

rows I and 2.

( Property 1)

b.) 
$$\begin{vmatrix} x & y & z \\ 2 & 4 & 6 \\ u & v & w \end{vmatrix} = - 8$$

They multiplied row 3 by 2 and then swapped rows

2 and 3

Prop 4 says Value of Letern. Will change by a factor of K

(K=Q)

Says interdranging

rows will cause value of det to change sign.