Hey, what's that?

College algebra
Class notes
Matrices: Determinants (section 8.3)

Determinants can be used to solve systems of equations by using something called **Cramer's Rule** (sometimes, but *not* always). There are other uses like for certain integration problems in calculus and certain geometric calculations but we won't go there.

The process of finding a determinant for a matrix gives us a single value. (It is actually a function since you get only one determinant for a given matrix.) We will play around with this idea to get our toes wet but not go much further. There is a fairly simple calculation, at least for 2x2 matrices. After that, it gets ugly… quite ugly… and we will use the calculator.

Formula for the Determinant of a 2x2 Matrix:

Define a square matrix $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$. Here, a, b, c, and d are real numbers. The determinant of this matrix is calculated as $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$.

Notice the difference between a matrix's brackets and its determinant's brackets.

expl 1: Find the determinant below.

$$\begin{vmatrix} 3 & 2 \\ -1 & 5 \end{vmatrix} = 15 - {}^{-}2 = 17$$

expl 2: Find the determinant below.

$$\begin{vmatrix} 7 & 14 \\ 2 & 4 \end{vmatrix} = 28 - 28 = 0$$

Finding Determinants on the Calculator:

You must enter a square matrix (same number of rows and columns). Otherwise, it will give you an error.

1. Enter the **MATRIX** menu. It is the **2nd** function of the $\mathbf{x^{-1}}$ button. Arrow over to **EDIT**.
2. Select **[A]** and press **ENTER**. You will then enter the order of the matrix, rows x columns.
3. Fill in the matrix with its entries.
4. Quit out to the home screen and re-enter the **MATRIX** menu. This time arrow over to **MATH**. Select **1:det(**. This is at the top of the list.
5. Re-enter the **MATRIX** menu and select **[A]** from the **NAMES** list.
6. Your home screen should now read **det([A]**. Press **ENTER** and it will output the matrix's determinant.

Formula for Determinant of 3x3 Matrix:

Nah, I'm joking. We aren't doing that by hand. Do those with the calculator.

expl 3: Find the determinant below.

$$\begin{vmatrix} 2 & 3 & -1 \\ 5 & 4 & -6 \\ 8 & 2 & -3 \end{vmatrix} = -77$$

$\begin{vmatrix} 3 & 2 \\ -1 & 5 \end{vmatrix} = 17$ (from pg 1)

Properties of Determinants (or, What Some Smart People Learned Long Ago About Determinants):

1. If any two rows (or any two columns) of a matrix are interchanged, then the value of the determinant changes sign.

Ooh, weird! Look back on example 1 and then find $\begin{vmatrix} -1 & 5 \\ 3 & 2 \end{vmatrix}$. $\rightarrow -2-15 = -17$

2. If all the entries in any row (or any column) equal 0, the value of the determinant is 0.

Aha! Make up such a 2x2 matrix and find its determinant. $\rightarrow \begin{vmatrix} 4 & -3 \\ 0 & 0 \end{vmatrix} = 0-0 = 0$

OR $\begin{vmatrix} 5 & 0 \\ 7 & 0 \end{vmatrix} = 0-0 = 0$

3. If any two rows (or any two columns) of a determinant have corresponding entries that are equal, the value of the determinant is 0.

Let's prove this one! Find the determinants $\begin{vmatrix} a & b \\ a & b \end{vmatrix}$ and $\begin{vmatrix} a & a \\ b & b \end{vmatrix}$.

$\begin{vmatrix} a & b \\ a & b \end{vmatrix} = ab - ab = 0$ ✓

$\begin{vmatrix} a & a \\ b & b \end{vmatrix} = ab - ab = 0$ ✓

4. If any row (or any column) of a determinant is multiplied by a non-zero number k, then the value of the determinant is also changed by a factor of k.

Ooh, weird! Look back on example 1 and then find $\begin{vmatrix} 3 & 2k \\ -1 & 5k \end{vmatrix}$. Pick a specific non-zero number for k or find this determinant with the variable k in place.

$\rightarrow \begin{vmatrix} 3 & 2k \\ -1 & 5k \end{vmatrix}$

$= 15k + 2k$

$= 17k$ ✓

Determinant Property Number 5 (so juicy it needed its own page):

5. If the entries of any row (or any column) of a determinant are multiplied by a non-zero number k and the result is added to the corresponding entries of another row (or column), then the value of the determinant remains unchanged.

So, $\begin{vmatrix} 3 & 2 \\ -1 & 5 \end{vmatrix} = 17$. Let's multiply row 2 by 2 and add it to row 1, getting $\begin{vmatrix} 1 & 12 \\ -1 & 5 \end{vmatrix}$.

What is this new determinant?

$\begin{vmatrix} 3 & 2 \\ -1 & 5 \end{vmatrix}$ row 2 mult by 2 $\rightarrow$ $-2 \quad 10$

and add it to row 1

getting $\begin{vmatrix} 1 & 12 \\ -1 & 5 \end{vmatrix}$

$= 5 - -12 = 17$

expl 4: It is known that $\begin{vmatrix} x & y & z \\ u & v & w \\ 1 & 2 & 3 \end{vmatrix} = 4$. Find the values of the following determinants using their properties discussed above. Give a brief explanation; which property(s) are you using?

a.) $\begin{vmatrix} u & v & w \\ x & y & z \\ 1 & 2 & 3 \end{vmatrix} = -4$

Because we interchanged rows 1 and 2.

(Property 1)

b.) $\begin{vmatrix} x & y & z \\ 2 & 4 & 6 \\ u & v & w \end{vmatrix} = -8$

They multiplied row 3 by 2 and then swapped rows 2 and 3.

Prop 4 says value of determ. will change by a factor of k ($k = 2$).

Prop 1 says interchanging rows will cause value of det. to change sign.