

Here are some real-life applications of the exponential function.

We are going to invest an amount of money (**principal, present value**) into a bank account that pays **compound interest**. We will find the amount of money at the end of some time period (**accumulated value**). Sometimes, we frame this as someone **borrow**s  $P$  dollars and is charged compound interest.

You may have studied **simple interest** in the past.

Consider a principal of \$1,000 invested at 5% *simple interest* for 4 years. The money earns interest at the rate of 5% of the initial deposit every year for 4 years. Hence,  $I = PRT = 1000 \times .05 \times 4 = \$200$ . After four years, they have a total of \$1,200.

With simple interest, interest is based on how much was initially deposited. But after one year, we have an extra \$50 (the interest earned or 5% of \$1,000) that we could invest. *If* we put that money back into the account and let it earn interest, we call that **compound interest**. All interest that is earned is subsequently reinvested to start earning the same rate as the initial deposit. The formula for that is here.

**Compound Interest Formula:**

$$A = P \left( 1 + \frac{r}{n} \right)^{nt}$$

$P$  = initial deposit (principal)  
 $A$  = amount in account after  $t$  years  
 $r$  = annual interest rate (decimal form)  
 $n$  = number of compoundings per year  
 $t$  = number of years

This assumes the account is compounded in one of several common ways. We could compound an account yearly ( $n = 1$  time a year), semiannually ( $n = 2$  times a year), quarterly ( $n = 4$ ), monthly ( $n = 12$ ), weekly ( $n = 52$ ), or daily ( $n = 365$ ).

And then, there's compounding continuously. Here, we imagine that  $n$  gets larger and larger; that is,  $n \rightarrow \infty$ . The formula actually becomes simpler.

**Continuously Compounded Interest Formula:**

$$A = Pe^{rt}$$

The variables mean the same but notice we are using the irrational number  $e$ .

expl 1: Use the compound interest formula to answer the following questions.

a.) Suppose \$150,000 is deposited in an account that pays 3.4% compound interest, compounded quarterly. Find the function for the amount  $A(t)$  in the account after  $t$  years.

$$A = P \left( 1 + \frac{r}{n} \right)^{n \cdot t}$$

$P$  = initial deposit

$A$  = amount in account after  $t$  years

$r$  = interest rate (decimal form)

$n$  = number of compoundings per year

$t$  = number of years

b.) How much is in the account after 4.5 years?

### Solving for Present Value:

expl 2a: Solve the continuously compounded interest formula for  $P$ .

$$A = Pe^{r \cdot t}$$

This will be used when we know how much we want in so many years but do *not* know how much to invest now.

**For non-continuously compounded accounts:**

$$P = A \left( 1 + \frac{r}{n} \right)^{-n \cdot t}$$

expl 2b: Margot needs to accumulate \$50,000 in ten years. How much should she deposit now in an account that pays 2.3% interest compounded continuously?

**Solving for Present Value Again:**

We will use the formula  $P = A \left( 1 + \frac{r}{n} \right)^{-nt}$  to find the present value (principal) needed to invest now in order to accumulate a certain amount ( $A$ ) in the future.

expl 3: Find the principal needed now to accumulate \$3,000 on 3.5 years if the account earns 1.25% compounded daily.

**Solving for Time:**

We will see that the variable we are solving for is in the exponent position. This will require our knowledge of solving exponential equations acquired earlier.

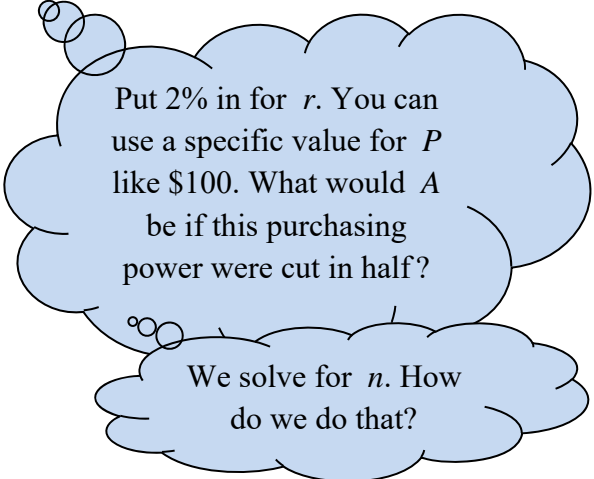
expl 4: Marcos wants \$2500 to go on vacation. He has \$1500 he will invest in an account that will compound continuously at 5% to earn his vacation fund. How many years (to the nearest tenth) will it take?

**Inflation:**

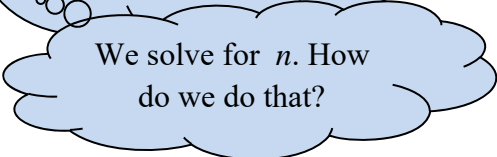
Inflation means the erosion of purchasing power of money. For example, if the annual inflation rate is 3%, then \$100 worth of purchasing power now will have only \$97 worth of purchasing power in a year.

If the rate of inflation averages  $r\%$  per year over  $n$  years, the amount  $A$  that  $P$  dollars will purchase after  $n$  years is  $A = P(1-r)^n$ .

expl 5: If the average inflation rate is 2%, how long will it be until purchasing power is cut in half?



Put 2% in for  $r$ . You can use a specific value for  $P$  like \$100. What would  $A$  be if this purchasing power were cut in half?



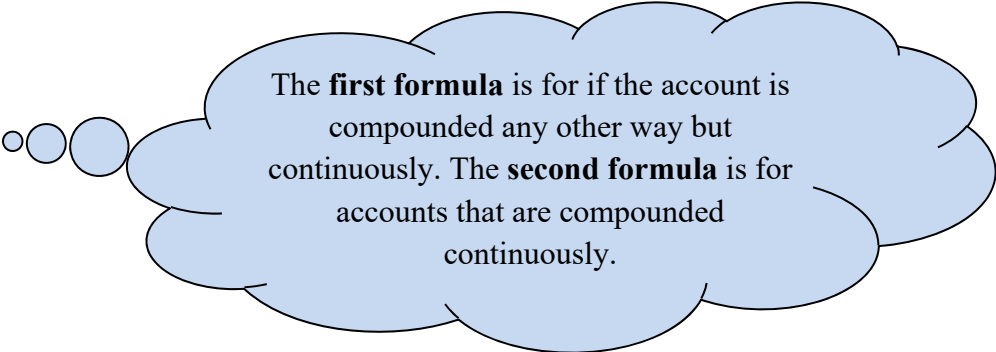
We solve for  $n$ . How do we do that?

**Effective Rate of Interest:**

The **effective rate of interest** ( $r_E$ ) is the annual simple interest rate that would yield the same amount as compounding  $n$  times per year, or continuously, after 1 year. Our formulas are below.

$$r_E = \left(1 + \frac{r}{n}\right)^n - 1$$

$$r_E = e^r - 1$$



The **first formula** is for if the account is compounded any other way but continuously. The **second formula** is for accounts that are compounded continuously.

expl 6: Find the effective rate of interest for the following account. Round to the nearest thousandths of a percent.

*5% compounded quarterly*