Here are some real-life applications of the exponential function.

College algebra
Class notes
Financial Models (section 6.7)

We are going to invest an amount of money (**principal**, **present value**) into a bank account that pays **compound interest**. We will find the amount of money at the end of some time period (**accumulated value**). Sometimes, we frame this as someone **borrows** P dollars and is charged compound interest.

You may have studied **simple interest** in the past.

Consider a principal of $1,000 invested at 5% *simple interest* for 4 years. The money earns interest at the rate of 5% of the initial deposit every year for 4 years. Hence, $I = PRT = 1000 \times .05 \times 4 =$ $200. After four years, they have a total of $1,200.

With simple interest, interest is based on how much was initially deposited. But after one year, we have an extra $50 (the interest earned or 5% of $1,000) that we could invest. *If* we put that money back into the account and let it earn interest, we call that **compound interest**. All interest that is earned is subsequently reinvested to start earning the same rate as the initial deposit. The formula for that is here.

Compound Interest Formula:

$$A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

P = initial deposit (principal)
A = amount in account after t years
r = annual interest rate (decimal form)
n = number of compoundings per year
t = number of years

This assumes the account is compounded in one of several common ways. We could compound an account yearly (n = 1 time a year), semiannually (n = 2 times a year), quarterly (n = 4), monthly (n =12), weekly (n = 52), or daily (n = 365).

And then, there's compounding continuously. Here, we imagine that n gets larger and larger; that is, $n \to \infty$. The formula actually becomes simpler.

Continuously Compounded Interest Formula:

$$A = Pe^{r \cdot t}$$

The variables mean the same but notice we are using the irrational number e.

expl 1: Use the compound interest formula to answer the following questions.

a.) Suppose \$150,000 is deposited in an account that pays 3.4% compound interest, compounded quarterly. Find the function for the amount $A(t)$ in the account after t years.

$$A = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$

P = initial deposit
A = amount in account after t years
r = interest rate (decimal form)
n = number of compoundings per year
t = number of years

b.) How much is in the account after 4.5 years?

Solving for Present Value:

expl 2a: Solve the continuously compounded interest formula for P.

$$A = Pe^{r \cdot t}$$

This will be used when we know how much we want in so many years but do *not* know how much to invest now.

For non-continuously compounded accounts:

$$P = A\left(1 + \frac{r}{n}\right)^{-n \cdot t}$$

expl 2b: Margot needs to accumulate \$50,000 in ten years. How much should she deposit now in an account that pays 2.3% interest compounded continuously?

Solving for Present Value Again:

We will use the formula $P = A\left(1+\frac{r}{n}\right)^{-n\cdot t}$ to find the present value (principal) needed to invest now in order to accumulate a certain amount (*A*) in the future.

expl 3: Find the principal needed now to accumulate $3,000 on 3.5 years if the account earns 1.25% compounded daily.

Solving for Time:
We will see that the variable we are solving for is in the exponent position. This will require our knowledge of solving exponential equations acquired earlier.

expl 4: Marcos wants $2500 to go on vacation. He has $1500 he will invest in an account that will compound continuously at 5% to earn his vacation fund. How many years (to the nearest tenth) will it take?

Inflation:

Inflation means the erosion of purchasing power of money. For example, if the annual inflation rate is 3%, then $100 worth of purchasing power now will have only $97 worth of purchasing power in a year.

If the rate of inflation averages r % per year over n years, the amount A that P dollars will purchase after n years is $A = P(1-r)^n$.

expl 5: If the average inflation rate is 2%, how long will it be until purchasing power is cut in half?

Put 2% in for r. You can use a specific value for P like $100. What would A be if this purchasing power were cut in half?

We solve for n. How do we do that?

Effective Rate of Interest:

The **effective rate of interest (r_E)** is the annual simple interest rate that would yield the same amount as compounding n times per year, or continuously, after 1 year. Our formulas are below.

$$r_E = \left(1+\frac{r}{n}\right)^n - 1$$

$$r_E = e^r - 1$$

The **first formula** is for if the account is compounded any other way but continuously. The **second formula** is for accounts that are compounded continuously.

expl 6: Find the effective rate of interest for the following account. Round to the nearest thousandths of a percent.

5% compounded quarterly