College algebra
Transformations
Section 3.5

We will move functions to the left, right, up, and down. We will squeeze and stretch them too.

Library of Functions:

Draw from memory or use your calculator (on the Standard window) to graph the following functions. You should acquaint yourself with their basic shapes.

Identity function $y = x$	**Square function** $y = x^2$	**Square root function** $y = \sqrt{x}$
Cube function $y = x^3$	**Cube root function** $y = \sqrt[3]{x}$	
Absolute value function $y = \lvert x \rvert$	**Reciprocal function** $y = \dfrac{1}{x}$	

Record domains, ranges, and intercepts.

These functions will serve as base or (I am *not* making this up!) mother functions. We will study how to transform these graphs by shifting, reflecting, stretching, and compressing (also called shrinking or squashing) the graphs.

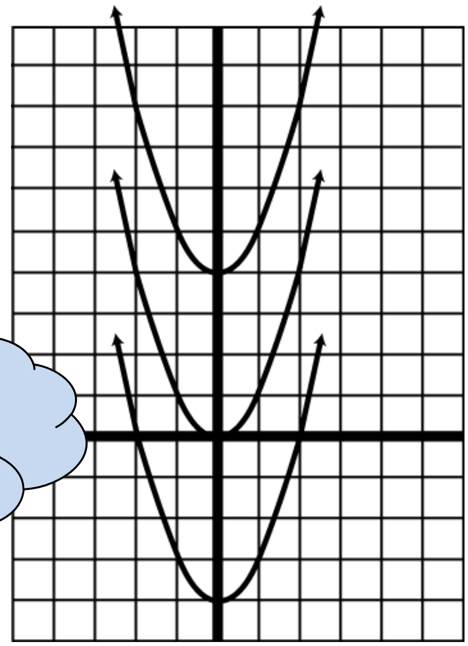
expl 1: Graph the following on your calculator, using the Standard window.

$y = x^2$

$y = x^2 - 4$

$y = x^2 + 4$

Apply the function, **then** add or subtract a number …



The graphs are shown here. Assume a scale of one unit per box. Can you tell which function is which?

How are the second and third graphs related to the first?

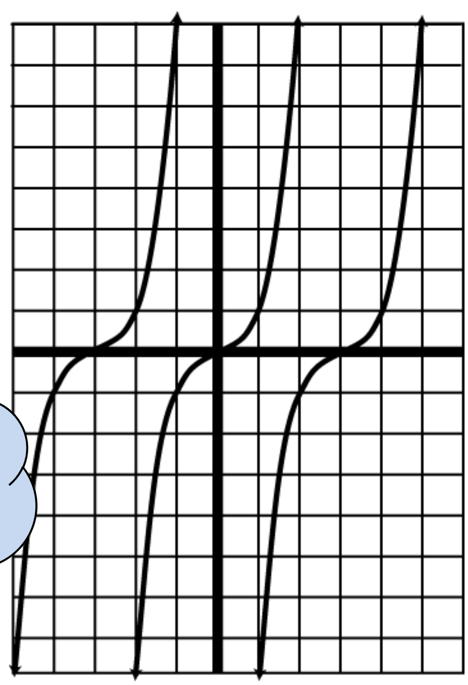
expl 2: Graph the following on your calculator, using the Standard window.

$y = x^3$

$y = (x - 3)^3$

$y = (x + 3)^3$

Add or subtract a number, **then** apply the function …



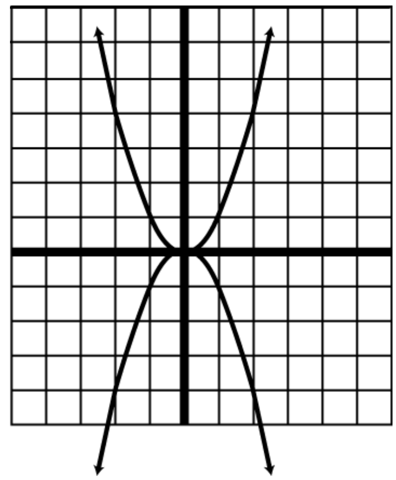
The graphs are shown here. Assume a scale of one unit per box. Can you tell which function is which?

How are the second and third graphs related to the first?

expl 3a: Graph the following on your calculator, using the Standard window.

$y = x^2$

$y = -x^2$

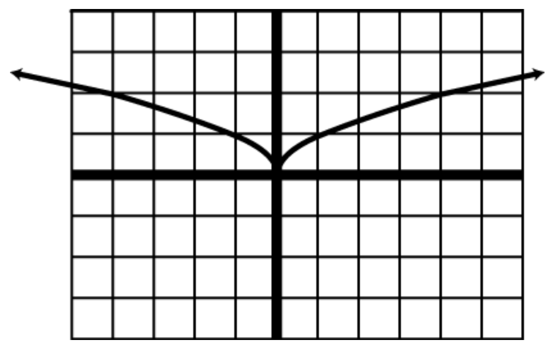


The graphs are shown here. Assume a scale of one unit per box. Can you tell which function is which?

expl 3b: Graph the following on your calculator, using the Standard window.

$y = \sqrt{x}$

$y = \sqrt{-x}$



The graphs are shown here. Assume a scale of one unit per box. Can you tell which function is which?

How would you describe these two transformations? What about the formulas make them different?

Do we apply the mother function or multiply by -1 first?

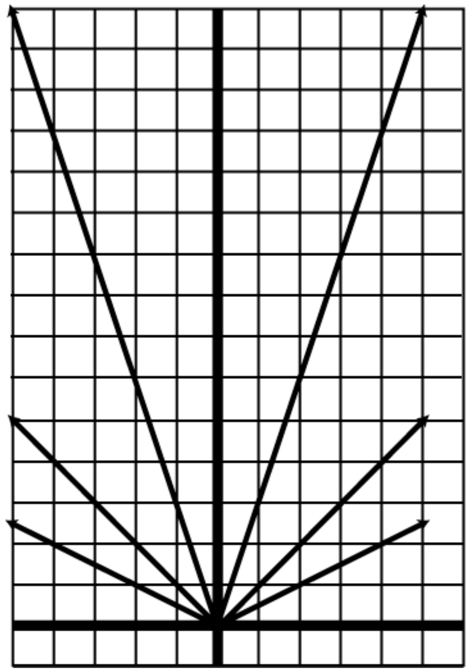
expl 4: Graph the following on your calculator, using the Standard window.

$y = |x|$

$y = 3|x|$

$y = .5|x|$

Apply the function, **then** multiply by a factor…



The graphs are shown here. Assume a scale of one unit per box. Can you tell which function is which?

How are the second and third graphs related to the first?

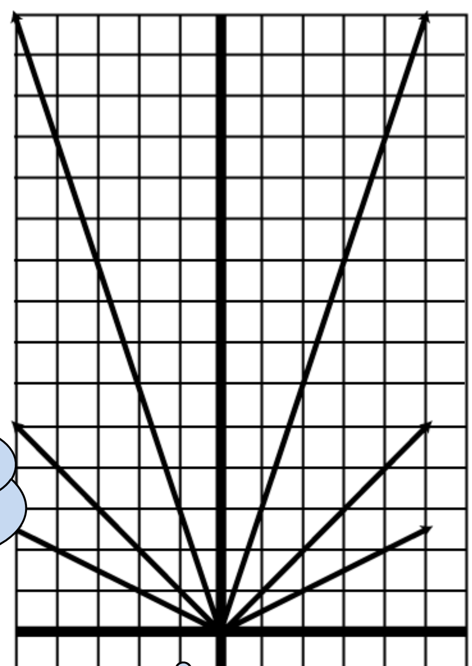
expl 5: Graph the following on your calculator, using the Standard window.

$y = |x|$

$y = |3x|$

$y = |.5x|$

Multiply by a factor, **then** apply the function …



The graphs are shown here. Assume a scale of one unit per box. Can you tell which function is which?

How are the second and third graphs related to the first?

The graphs in numbers 4 and 5 look identical, don't they? Only the formulas tell you they are different transformations.

Review: Function Notation:

How good at using function notation are you? Try this problem out.

expl 6: For the function $f(x) = x^2$, find the following. Do *not* simplify.

a.) $f(x-3)$ b.) $f(x)+4$ c.) $f(3x)$

Transformations Summary:

Example	**Transformation**	Let $f(x)$ be the original or base function. We call x the argument. Let c be a real positive number.
1	Vertical shift down c units	$f(x)-c$
	Vertical shift up c units	$f(x)+c$
2	Horizontal shift to right c units	$f(x-c)$
	Horizontal shift to left c units	$f(x+c)$
3a	Reflection about x-axis	$-f(x)$
3b	Reflection about y-axis	$f(-x)$
4	Vertical stretch by a factor of c	$c * f(x), \quad c>1$
	Vertical compression (shrink) by a factor of c	$c * f(x), \quad 0<c<1$
5	Horizontal compression (shrink) by a factor of $1/c$	$f(c \cdot x), \quad c>1$
	Horizontal stretch by a factor of $1/c$	$f(c \cdot x), \quad 0<c<1$

Other books may phrase these last two "by a factor of c".

A closer look:
For a better understanding of how these transformations work, complete the table for the two functions below. Similar tables can be made to look at other transformations.

x	-3	-2	-1	0	1	2	3
$f(x) = x^2$							
$h(x) = x^2 + 3$							

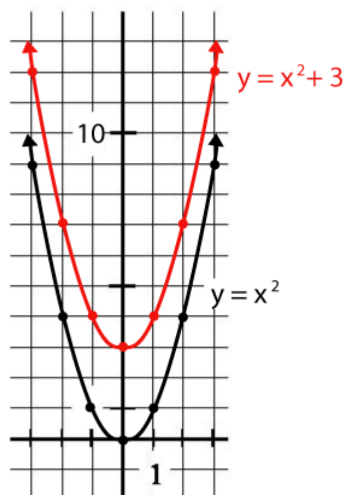
How are the y values of $h(x)$ related to the y values of $f(x)$? What effect does that have on the graph of $h(x)$?

I have drawn the graphs of $f(x) = x^2$ and $h(x) = x^2 + 3$ to the right.

The table values show that the $h(x)$ values are simply 3 more than each $f(x)$ value.

Do you see why this moves the graph of $f(x)$ **up** 3 units to form the graph of $h(x)$?

You can think through transformations by imagining what is happening to the y values.



For instance, if you compare $y_1 = x^2$ and $y_2 = 5x^2$, you'll notice the second function's y values are merely five times the first function's y values. So each point is stretched away from the x-axis. The graph will be elongated. So we say the graph is vertically stretched by a factor of 5. Check it out on your calculator!

As another example, consider $y_1 = x^3$ and $y_2 = (x-3)^3$. Here is a table of some values to ponder. Do you see why the graph of y_2 would be to the right of the graph of y_1?

x	-6	-3	0	3	6
$y_1 = x^3$	-216	-27	0	27	216
$y_2 = (x-3)^3$	-729	-216	-27	0	27

Worksheet: Transformations 2:
This worksheet gives you practice graphing these functions. You use tables of x and y values to help see why transformed functions behave as they do. You will also practice naming the transformations using the proper names.

expl 7: Describe how the graph of the following functions can be obtained from one of the functions listed in the Library of Functions at the beginning of the notes. Graph them on your calculator to verify.

a.) $g(x) = 5x^3$

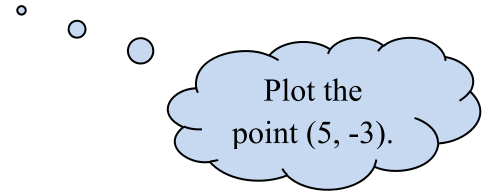
b.) $h(x) = |3x| - 2$

What are the domains and ranges of these functions?

c.) $g(x) = -4 \cdot \sqrt[3]{x} + 7$

expl 8: Let (5, -3) be a point on the graph of $f(x)$. Find the corresponding point on the graph of the functions labeled y below.

a.) $y = f(x) + 6$



b.) $y = \frac{1}{2} f(x)$

expl 9: Write an equation for the function with the following descriptions.

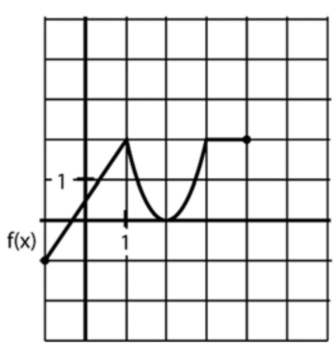
a.) The graph of $y = \frac{1}{x}$ but shifted to the left 4 units.

b.) The graph of $y = \sqrt{x}$ but vertically stretched by a factor of 3 and reflected across the x-axis.

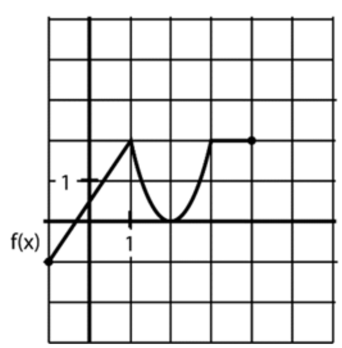
c.) The graph of $y = |x|$ but horizontally compressed by a factor of ½ and shifted up 5 units.

expl 10: Given the following graph of a function $f(x)$, graph the following functions.

a.) $f(x) + 2$



b.) $-f(x)$



c.) $f(x - 3)$