## College algebra

Class notes


Properties of Logarithms (section 6.5)

## Optional Worksheet: Logarithm Rules Worksheet

This worksheet will show you how to derive most of the formulas given in this section. If we can think through the formulas, they will be easier to memorize and apply. Try to use what you know about logs to figure out the following excerpt from the worksheet.

1. In words, what is $\log _{b} b$ ? It's the number to which I raise $\qquad$ to get $\qquad$ .

What does this number we call $\log _{b} b$ have to be?
2. In words, what is $\log _{b} 1$ ? It's the number to which I raise $\qquad$ to get $\qquad$ .

What does this number we call $\log _{b} 1$ have to be?
3. In words, what is $\log _{b} b^{k}$ ? It's the number to which I raise $\qquad$ to get $\qquad$ .

What does this number we call $\log _{b} b^{k}$ have to be?
4. Now $\log _{b} v$ is the number to which I raise $b$ to get $v$. So, when we raise $b$ to this number, what should I get? In other words, what is $b^{\log _{b} v}$ ?

The complete list of rules is listed below including the Change of Base Formula from the end of this section.

1. $\log _{a}(M / N)=\log _{a} M-\log _{a} N$
2. $\log _{a}(M \cdot N)=\log _{a} M+\log _{a} N$
3. $\log _{a} M^{r}=r \cdot \log _{a} M$
4. $\log _{a} a=1$

5. $\log _{a} 1=0$
6. $\log _{a} a^{r}=r$
7. $a^{\log _{a} M}=M$
8. $a^{r}=e^{r \ln a}$
9. $\log _{a} M=\frac{\log _{b} M}{\log _{b} a}$

## Common Mistakes:

It is common to incorrectly assume other rules similar to those given. Be careful when you apply the rules. You should also try out numbers in any rule you "think" is right.

expl 2: Express as a sum or difference of logs. Express powers as factors. Simplify if possible. $\ln y^{5}$
expl 3: Use the properties of logarithms to find the exact value of this expression.
$\log _{8} 2+\log _{8} 4$
expl 4: Simplify.
а.) $t^{\log _{t} 3}$
b.) $\ln e^{6}$
expl 5: Express as a sum or difference of logs. Express powers as factors. Simplify if possible.
a.) $\log \frac{x^{2} y}{(x+1)}$

b.) $\log _{5} \frac{(x+2)^{3}}{x^{4}}$
expl 6: Express as a single log. Express powers as factors. Simplify if possible.
a.) $\ln \left(\frac{x}{x-2}\right)-\ln \left(x^{2}-4\right)+\ln \left(\frac{x+2}{x}\right)$

b.) $3 \cdot \ln (x-5)-[\ln (x-5)+\ln (x+5)]$


## Change of Base Formula:

You cannot find logs other than base 10 or $e$ on your calculator. We need another way. For any logarithmic bases $a$ and $b$, and any positive number $M$, we know that $\log _{b} M=\frac{\log _{a} M}{\log _{a} b}$.

[We will usually use 10 or $e$ for the base $a$ so we can do these problems on the calculator.]
expl 7: Use the Change of Base Formula and your calculator to find the following. $\log _{3} 12$

expl 8: Use the Change of Base Formula to rewrite the following and then evaluate without a calculator.
$\log _{3} 5 \cdot \log _{5} 81$
expl 9: Graph on a calculator using the Change of Base Formula. Use a window of $[-2,5] \times[-3,3]$.
$y=\log _{4} x$


## Solving Special Logarithmic Equations:

What would you say the solution to $\log _{2} x=\log _{2} 16$ is? Go with your gut. There is actually a pretty simple property that backs up what your gut probably told you.

Logarithmic Equality Property: For any $M>0, N>0, a>0$, and $a \neq 1$, we know that $M=N$ if and only if $\log _{a} M=\log _{a} N$.

expl 10: Express $y$ as a function of $x$. The constant $C$ is a positive number. $\log y=2 \cdot \log x-\log C$

expl 11: It is easy to remember these rules incorrectly. We may need to check our memory. Substitute values for the variables to check if the following "rule" is true. State your conclusion specifically. If it is not true, be sure to strike through the equal sign. Use a base of 10 or $e$ so you can use your calculator to evaluate the logs.
$\frac{\log _{a} M}{\log _{a} N} \stackrel{?}{?} \log _{a} M-\log _{a} N$

