Solving a system of equations means to find the point that makes **all** of the equations true.

College algebra
Class notes
Matrices: Solving Systems of Linear Equations in Two or Three Variables (section 8.1)

To solve a system of equations means to find the one point that satisfies all equations. We will learn three methods, one graphical and two algebraic methods, that are used to solve systems of two or three equations. This will give us an understanding for how we solve these systems using matrices later.

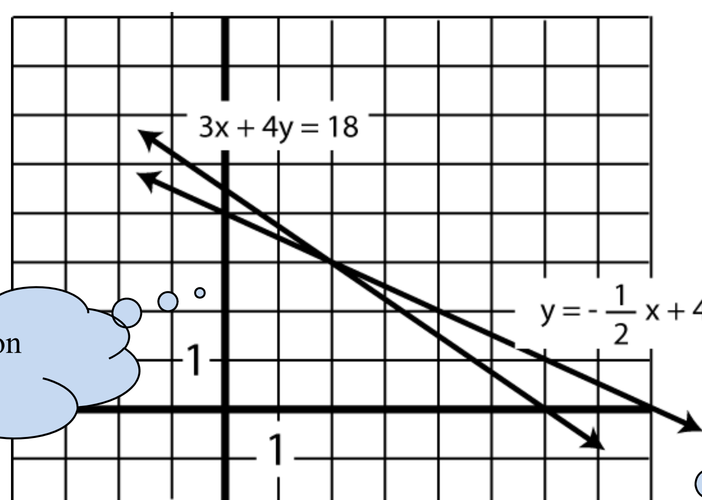
Graphically Solving Systems of Linear Equations:
This method uses the facts that …

the graph of a linear equation is always a straight line, and
all the points on the graph of a line satisfy the equation (that is, makes it true).

Now, if we want to find the point that satisfies two different equations, we would start by graphing both lines. Then what would we look for on the graph? See the example below.

expl 1: Solve the system. The two lines are graphed for you.

$$3x + 4y = 18$$
$$y = -\frac{1}{2}x + 4$$



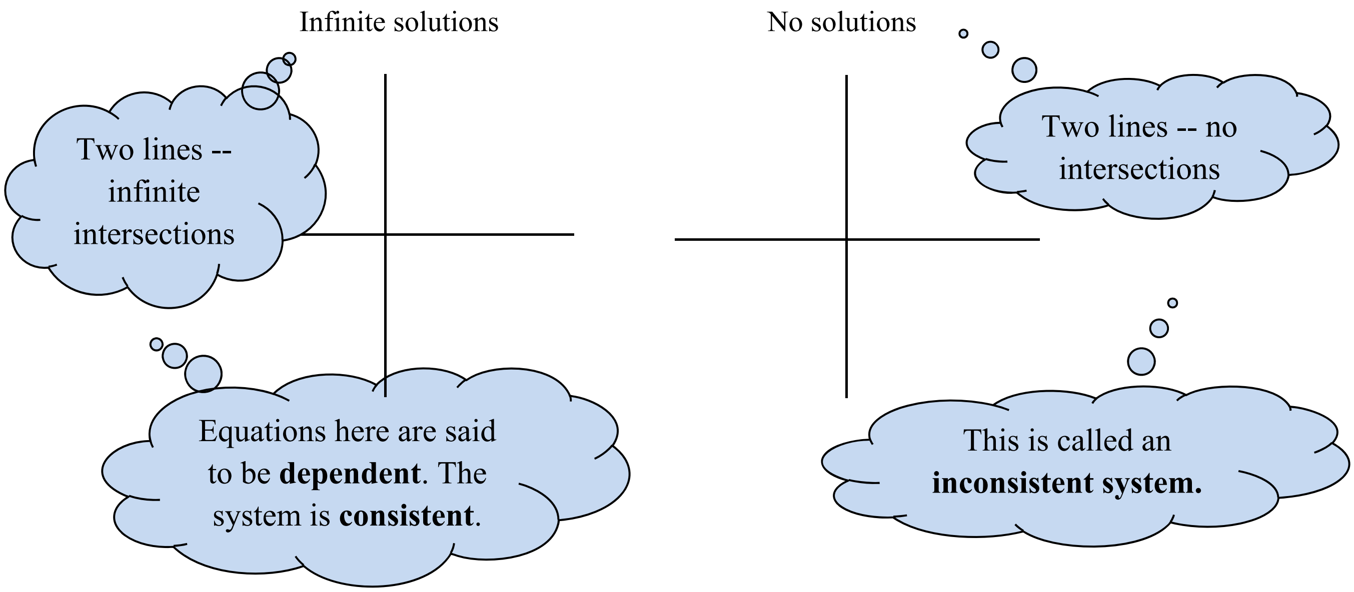
What is the solution to the system?

Can you graph lines in MML?

Try your solution in both equations. Does it make **both** equations true?

The system of equations above is called **consistent** and the equations (since they have only one intersection) are called **independent**.

Systems with infinitely many or no solutions:
So, the solution to a system of equations is the intersection of the two lines. The preceding example has one solution (corresponding to one intersection). There are two other possibilities, an infinite number of solutions or no solutions. Think about how those systems would look and draw possible graphs below.



Is it possible for two lines to intersect in exactly two or three points? Explain.

Optional Worksheet: Solving systems of equations graphically: Calculator worksheet:

This worksheet shows solving equations for y in order to put them into the calculator, using the Intersect function on the calculator (TI-82, 83, 84, 85, and 86) to find the exact intersection, and finding an appropriate window for our graphs.

Algebraically Solving Systems of Linear Equations by Substitution:
Now, we address this problem algebraically. Consider our dilemma. We are given two equations but we can't solve either for x because that darn y is in the way. If only we had just one equation with just one variable. Then we could solve for that variable like we are used to. Look at the example here.

expl 2: Solve the system by substitution.

$$3x + 4y = 18$$
$$y = -\frac{1}{2}x + 4$$

> How do you turn these two equations with two variables into one equation with one variable? Substitution, baby!

> Don't forget about y. Write your solution as an ordered pair.

Algebraically Solving Systems of Linear Equations by Elimination (or Addition):
We will simply add the equations. This will eliminate one of the variables. We then solve for the variable that's left over. Remember to write your solution as an ordered pair.

expl 3: Solve the system by elimination.

$$3x + 2y = 2$$
$$5x - 2y = 14$$

> Add the left sides, add the right sides, and set them equal. Watch the magic happen!

Adding now will *not* eliminate a variable. How can we change one of the equations so that x will be eliminated when we add?

expl 4: Solve the system by elimination.

$$x + 4y = 14$$
$$5x + 3y = 2$$

expl 5: Solve algebraically.

$$\frac{1}{4}x - 2y = 1$$
$$x - 8y = 4$$

What was the solution last time we got a result like this?

The system above has infinite solutions. We have to write the points that satisfy both equations in ordered pair notation just like before. Solve one of the equations for y and then write the solution as $\{(x, y) \mid y = \qquad , x \textit{ any real number}\}$. Similarly, we could solve one of the equations for x and then write the solution as $\{(x, y) \mid x = \qquad , y \textit{ any real number}\}$. Do it now.

expl 6: Solve algebraically.

$$\begin{aligned} -3x + y &= 7 \\ -6x + 2y &= -8 \end{aligned}$$

What was the solution last time we got a result like this? What do we call this system?

Systems of Three Equations:

Systems of two equations with two unknowns represent lines. Systems of three equations with three unknowns represent planes in 3D space. Here, we are finding the intersection(s) of three planes. We will solve these with matrices later. But let’s get our toes wet.

expl 7: Determine if the values of the variables comprise a solution to the system of equations given.

$$\begin{aligned} 3x + 3y + 4z &= 16 \\ x - y - z &= 0 \\ 3y - 2z &= -14 \end{aligned}$$

$x = 2$, $y = -2$, $z = 4$ (This is the point (2, –2, 4) in 3D space.)

Does the point make every equation true?

Systems of equations may have one intersection, no intersection, or infinitely many intersections. We will next see how we can use matrices to shortcut the algebra of solving these systems. This is particularly important when we solve systems with three or more equations.